Forward Guidance with Bayesian Learning and Estimation

Christopher Gust Edward Herbst David López-Salido*

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Abstract

We estimate a New Keynesian model in which the private sector has incomplete information about a central bank's reaction function and must infer it based on economic outcomes. A central bank's reaction function can change across regimes and we document a systematic change in U.S. policymakers' reaction function during the 2009-2016 period in which the federal funds rate was at the effective lower bound. This regime calls for a persistently lower policy rate than the other regimes and, all else equal, has the effect of extending the duration that the policy rate remains at the zero lower bound (ZLB); hence, we call this the forward guidance regime. Our estimates suggest that private sector agents were slow to learn about this change in real time, which limited the effectiveness of the forward guidance regime in stimulating economic activity and curbing disinflationary pressure. Our analysis highlights the importance of incorporating incomplete information into structural models, as we show that the incomplete information specification of the model fits economic outcomes over the economy's long spell at the ZLB better than the full information specification.

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*Federal Reserve Board. christopher.gust@frb.gov, edward.herbst@frb.gov; david.lopez-salido@frb.gov. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

1 Introduction

With policy rates around the world falling to the zero lower bound (ZLB), many central banks looked for ways to provide additional policy accommodation during the global financial crisis. Communication about the future path of the policy rate, commonly referred to as forward guidance (FG), was an important part of the monetary policy strategy of overcoming the ZLB constraint and providing such stimulus. Through its influence on private-sector expectations of future short-term interest rates, forward guidance can lower longer-term interest rates and thereby help monetary policymakers achieve their objectives.

An important part of FG communications is the information policymakers provide to the public about their reaction function (i.e., how a policymaker intends to adjust the policy rate in response to economic developments.) At the ZLB, *a change* in the reaction function that keeps the policy rate lower for longer than implied by the previous reaction function can in principle stimulate aggregate demand and curb disinflationary pressure. However, for such a policy to be effective, the public must believe a policymaker's reaction function has changed and in practice it can be quite difficult for the public to perceive such a change. First, private sector agents do not observe a central bank's reaction function and must infer it. This inference becomes even more difficult at the ZLB where agents do not observe changes in the policy rate. Second, communications about future policy rates are open to other interpretations besides signaling a change in policymaker's reaction function. For instance, such communications may just provide clarity about a pre-existing reaction function or convey information about policymaker's forecasts of economic conditions instead of a commitment to keeping the policy rate lower for longer than they would have under their pre-existing reaction function. Given these considerations, open questions remain whether the FOMC's communications about FG implied a change in their reaction function and whether private agents were able to perceive such a change.

In this paper, we address these questions by estimating a New Keynesian (NK) model in which private sector agents have incomplete information about a policymakers' reaction function.[1] In the model, the central bank's interest-rate reaction function can change across regimes and agents do not observe these regimes but must infer them. A change in the policy regime is imperfectly credible, as agents' beliefs about the regime can differ from the true (newly-realized) regime. These beliefs evolve endogenously as agent learn from observing outcomes for interest rates, inflation, and other economic

[1] For early applications of models with incomplete information, see, for example, Lucas (1972) or Gertler (1982).

variables and use this information to update their prior beliefs about the regimes.

We model regime changes as affecting the intercept of an interest-rate rule. We prefer such an approach to other alternatives for two reasons. First, it aligns well with FG communications by the Federal Reserve in 2014-15. In particular, through most of 2014 and into early 2015 the Federal Open Market Committee indicated in its postmeeting statements that

> *The Committee currently anticipates that, even after employment and inflation are near mandate-consistent levels, economic conditions may, for some time, warrant keeping the target federal funds rate below levels the Committee views as normal in the longer run.*

In our view, such a statement is more supportive of a regime change that affects the intercept of a rule rather than a change involving the degree to which the rule responds to economic developments. Second, FG communications evolved over the 2009-2015 period, becoming more explicit and forceful over time. These communications were at times largely time dependent in the guidance they provided about future values of the federal funds rate and at other times more state dependent. Because of the evolving nature of the FOMC's FG language, in our view there is no unique way to capture FG and we view allowing for regime changes through the intercept term as a relatively agnostic way of doing so, because it imposes less structure on the reaction function than other approaches.

A novel feature of our model is the way in which the ZLB constraint affects the way agents learn about a central bank's reaction function. In particular, at the ZLB, it is even more difficult for agents to infer a central bank's reaction function, since they do not observe changes in the policy rate or a central bank's desired or notional rate. For rational agents updating their beliefs according to Bayes rule, we show that the ZLB constraint gives rise to a likelihood function that is a mixture of two distributions, similar to a Tobit model, because the ZLB truncates agent's observations of the central bank's desired policy rate. If agents observe the policy rate at the ZLB, they update their beliefs by determining the likelihood of each regime to lead to a binding ZLB constraint.

Our model allows us to assess the effectiveness of forward guidance on the macroeconomy and the importance of imperfect information in limiting its effectiveness — major goals of our empirical analysis. By estimating the model we provide new evidence documenting a change in Federal Reserve policy over the 2009-2016 period. In particular, our estimates indicate that there was a regime change in the FOMC's reaction function beginning in 2010 and that this new regime called for persistently lower interest rates than would have been the case if the FOMC had stuck to their pre-crisis reaction function. Hence, our estimates suggest that there was a change in the Federal Reserve's reaction function at the the time the Federal Reserve was providing FG communications and we call this new regime the FG regime.

While this regime change can be identified as early as 2010 by an econometrician who has the benefit of hindsight, agents have to make inferences about policy regimes in real time and thus can take longer to identify the change in policy regimes. Indeed, we find that agents only begin to put significant weight on the FG regime in 2012 and that it takes until 2014 before they become close to certain about being in the FG regime. We find that these slowly evolving beliefs are a direct implication of the nonlinear inference problem induced by the ZLB: agents no longer observe the desired rate and thus learn considerably more slowly than they would otherwise.

This slow evolution of beliefs has important implications for the effectiveness of the Federal Reserve's FG. With agents ascribing little weight to being in the FG regime until 2012, we estimate that the effects of FG on economic activity and inflation only begin to be realized until late 2012. Over time, the effects of FG increase, as agents come to realize that the Federal Reserve's reaction function changed. In 2014-15, when agents see over a 90% probability of being in the FG regime, our estimates imply that the change to the FG regime leads to about a percentage point increase in the output gap and about a 40 basis points increase inflation relative to a situation in which there was no change in the policy regime.

The estimation results also point to imperfect credibility as being a useful feature for explaining U.S. data over the period in which the Federal Reserve provided FG. By estimating the incomplete information (II) and full information (FI) specifications of the model, we show that II specification fits the data better than the FI specification. This better fit reflects that monetary policy regimes are imperfectly credible, and hence the II specification, unlike the FI specification, does not suffer from what Del Negro, Giannoni, and Patterson (2015) call the 'forward guidance puzzle' and does not deliver unreasonably large macroeconomic effects from the change to the FG regime.

We estimate the model using the methodology of Gust, Herbst, López-Salido, and Smith (2017), which is well suited to handling nonlinear models like ours in which the ZLB constraint occasionally binds. An advantage of this approach is that we can use all the data to estimate the model including episodes in which the policy rate is at or near the ZLB. Because the FOMC relied heavily on forward guidance when the economy was at the ZLB, it is important to use data during this period to identify and test whether there was a change in policymakers' reaction function. In contrast, Campbell, Fisher, Justiniano, and Melosi (2016) estimate their model using pre-crisis data and assume that the subsequent FG communications resulted in a change in the pre-crisis rule.

The methodology is also well suited to handling the nonlinearities introduced by the Bayesian learning of agents in the model. Private sector agents update their beliefs about policy regimes using Bayes rule, which is nonlinear even in the absence of the ZLB. In the absence of Bayesian learning, the model is standard in the literature and has been estimated by Gust, López-Salido, and Smith (2012) as well as Ireland (2011).[2]

The rest of this paper proceeds as follows. The next section discusses related literature, while Section 3 presents the model with a focus on how incomplete information about policy affects the expectations of private sector agents. Section 4 describes the econometric approach, and the empirical results are presented in Section 5. Section 6 concludes.

2 Related Literature

Our paper is most closely related to Engen, Laubach, and Reifschneider (2015) and Campbell, Fisher, Justiniano, and Melosi (2016), which both examine the effectiveness of the Federal Reserve's FG. Like Engen, Laubach, and Reifschneider (2015), we emphasize the importance of slowly evolving beliefs about changes in the reaction function for monetary policy in limiting the effectiveness of FG. However, we take a very different methodological approach by estimating a structural model and explicitly modeling the way in which private sector agents learn about a new policy regime. Like our paper, Campbell, Fisher, Justiniano, and Melosi (2016) estimate a structural model to study FG; however, they model FG using news shocks that are viewed by private sector agents as fully credible and thus do not study the role of imperfect credibility in limiting the effectiveness of FG.

Our paper is related to papers that emphasize learning or imperfect credibility in overcoming what Del Negro, Giannoni, and Patterson (2015) call the 'forward guidance puzzle' — a puzzle that applies to DSGE models in which future changes in monetary policy rate can deliver unreasonably large effects on macroeconomic variables (e.g., McKay, Nakamura, and Steinsson (2016)). Our paper is similar to

[2] We use a smaller model than in our previous work, Gust, Herbst, López-Salido, and Smith (2017), in order to keep the analysis with learning about policy regimes tractable. We view this as an important first step in incorporating learning about a central bank's reaction function into an estimated, medium-scale DSGE model.

De Graeve, Ilbas, and Wouters (2014) and Cole (2015) in that we introduce learning into a DSGE model in order to overcome the forward guidance puzzle. However, we differ from these papers, because we do not depart from rational expectations to introduce learning into our model and because we estimate our model. The latter is important in order to quantify the the effectiveness of FG on the macroeconomy and to assess role of imperfect credibility in an empirical setting.

Our incorporation of Bayesian learning into a monetary policy model builds on the analysis of Erceg and Levin (2003), Schorfheide (2005) and Svensson and Williams (2008). However, these papers do not incorporate the ZLB or study the effectiveness of forward guidance. Of these papers, our paper is most closely related to Schorfheide (2005), who, like us, estimates a structural model with regime changes for monetary policy in which private sector agents have imperfect information about the regimes. However, Schorfheide (2005) finds, using linearized models, that the full information specification fits the data over his sample period better than the incomplete information specification. In contrast, we find the II specification outperforms the FI specification, because we focus on monetary policy at the ZLB and show how the ZLB makes it more difficult for agents to discriminate across different policy regimes.

3 The Model

The economy consists of a representative household, a continuum of firms producing differentiated intermediate goods, a perfectly competitive final goods firm, and a central bank in charge of monetary policy.

3.1 Incomplete Information and Policy Uncertainty

Households and firms in the economy have imperfect information about monetary policy. In particular, the central bank follows an interest-rate reaction function that is regime-dependent and private sector agents do not observe the current policy regime. However, they observe the nominal interest rate, R_t, in period t, which satisfies:

$$\log R_t = \max\left[0, f_R\left(\mathbb{X}_t, \Gamma_{j_t}\right) + e_{R_t}\right], \tag{1}$$

where $f_R\left(\mathbb{X}_t, \Gamma_{j_t}\right)$ denotes the central bank's reaction function, $\mathbb{X}_t$ is a set of economic variables that is observable to private agents such as inflation and output, and Γ_{j_t} is a set of unobserved coefficients governing how responsive a central bank's desired policy rate is to economic developments in regime

$j_t \in \{1, 2, ..., N_J\}$. Households and firms also do not observe e_{R_t} — a temporary, exogenous deviation from that reaction function — though they know that it is normally distributed with zero mean and standard deviation, σ_R. They also do not observe j_t though they know that j_t follows a Markov process with the transition matrix, $\mathbf{P} = [P_{ij}]$, where P_{ij} denotes the probability of transitioning from regime i to regime j.

It is useful to define the desired policy rate as sum of the central bank's reaction function and the temporary deviation from that function, $f_R(\mathbb{X}_t, \Gamma_{j_t}) + e_{Rt}$. According to equation (1), when the policy rate is above zero, the desired rate coincides with the policy rate, $\log R_t$, though they can not distinguish between changes within the reaction reaction function and temporary deviations from it. When the policy rate is at the ZLB, agents can no longer observe the desired rate, which complicates their inference about the policy regime.

3.1.1 Bayesian Decision Making

With this information structure, agents face uncertainty about the current policy regime and must form beliefs about it. We follow the rational learning literature in which the agents are Bayesian decision makers. A change in the policy regime is imperfectly credible as agents' beliefs differ from the true regime. Agents enter the period with prior beliefs about the likelihood of each of the policy regimes and update their beliefs after observing R_t and $\mathbb{X}_t$ and use their knowledge of the model to judge the likelihood of each regime given these new observations. At the same time, households and firms make their optimal decisions in product, labor and asset markets. In doing so, they take into account their uncertainty about the policy regime on their decisions as well as the fact that their beliefs about monetary policy evolve over time.[3] Attractive features of our incomplete information approach are that it is fully model consistent and does not require us to specify additional model parameters relative to the complete information, rational expectations case.

To formalize this approach, let $p_{jt|t-1} \equiv \text{Prob}(j_t = j|\Omega_{t-1})$ denote the prior beliefs that households and firms have at the beginning of date t about regime j, where Ω_{t-1} corresponds to an agent's information set at time $t-1$. Importantly, Ω_{t-1} includes R_{t-1} and $\mathbb{X}_{t-1}$ but does not include j_{t-1} or e_{Rt-1} or any of their additional lags. At the same time as households and firms choose their optimal

[3] This differs from the 'anticipated utility' approach in which agents' beliefs are assumed not to change when they make their economic decisions. See Cogley and Sargent (2005) for a more thorough discussion of the difference between the Bayesian and 'anticipated utility' approaches to learning.

allocations, they update their beliefs about monetary policy taking on board their observations of R_t and $\mathbb{X}_t$. Agents do so using Bayes rule:

$$p_{jt|t} = \frac{\xi\left(R_t, \mathbb{X}_t | j_t = j\right) p_{jt|t-1}}{\xi_t' p_{t|t-1}}, \tag{2}$$

where $\xi\left(R_t, \mathbb{X}_t | j_t = j\right)$ denotes the likelihood of observing R_t and $\mathbb{X}_t$ conditional on policy regime j, $p_{t|t-1}$ denotes the vector, $(p_{1,t|t-1}, p_{2,t|t-1}, ..., p_{N_J,t|t-1})$, and ξ_t denotes the vector:

$$\xi_t = \left(\xi\left(R_t, \mathbb{X}_t | j_t = 1\right), \xi\left(R_t, \mathbb{X}_t | j_t = 2\right), ..., \xi\left(R_t, \mathbb{X}_t | j_t = N_J\right)\right).$$

Because an agent's observations of a central bank's desired policy rate are truncated at the zero lower bound, the conditional likelihood function of the observed data is similar to a Tobit model and is comprised of a mixture of two functions:

$$\xi\left(R_t, \mathbb{X}_t | j_t = j\right) = \left[\frac{1}{\sigma_R}\phi\left(\frac{\log R_t - f_R\left(\mathbb{X}_t, \Gamma_j\right)}{\sigma_R}\right)\right]^{\mathbb{I}_t}\left[1 - \Phi\left(\frac{f_R\left(\mathbb{X}_t, \Gamma_j\right)}{\sigma_R}\right)\right]^{1-\mathbb{I}_t}. \tag{3}$$

The functions ϕ and Φ are the pdf and cdf, respectively, for a standard normal. Also, $\mathbb{I}_t$ is an indicator variable satisfying $\mathbb{I}_t = 1$ if $\log R_t > 0$ and $\mathbb{I}_t = 0$ if $\log R_t = 0$.

When $\log R_t > 0$, equation (3) indicates that the relevant function for making inferences about the regimes is the normal density function for e_{Rt}. The function, ξ, is used to evaluate the likelihood of each regime, as agents use it to compare the observed policy rate to the central bank's reaction function under each regime. Conditional on $\log R_t > 0$, greater volatility in e_{Rt} will make it more difficult to determine the true regime and thus how monetary policy responds to the state of the economy. The conditional likelihood changes when $\log R_t = 0$. In this case, the term $1 - \Phi\left(\frac{f_R(\mathbb{X}_t, \Gamma_j)}{\sigma_R}\right)$ is the probability of observing a policy rate of zero conditional on being in regime j. Agents update their beliefs in this way because they do not observe the desired policy rate and instead judge a regime's likelihood by how it affects the probability of the policy rate being at the ZLB. As in the case in which $\log R_t > 0$, greater volatility in e_{Rt} makes it difficult to distinguish between policy regimes, since a larger, negative innovation in e_{Rt} is more likely, leading to a greater chance of being at the bound regardless of the regime.

Because agents are forward-looking, they need to form beliefs about future regimes. Accordingly, conditional on their beliefs about the current policy regime, agents form their priors about the regime in period $t + j$, $j > 0$, using

$$p_{t+j|t} = \mathbf{P}'^{j} p_{t|t}. \tag{4}$$

3.2 Households

The representative household chooses their consumption, C_t, labor supply, H_t, and asset allocations in financial markets taking into account its limited information about the policymaker's reaction function. The household chooses a state-contingent plan for consumption, hours, the risk-free bond, and equity shares to maximize:

$$\mathbb{E}\left\{\sum_{t=1}^{\infty}\beta^t\left(\log\left[C_t-\gamma C_{t-1}^a\right]-\chi_0 H_t+\eta_t\log\left[\frac{B_{t+1}}{P_t}\right]\right)|\Omega_0\right\}. \tag{5}$$

These preferences allow for external habits, as households value their own consumption relative to aggregate consumption, C_{t-1}^a in the previous period, which a household takes as given. The parameter $0\leq\gamma<1$ captures the importance of external habits. We include risk-free bonds in a household's utility function following Fisher (2015) and Krishnamurthy and Vissing-Jorgensen (2012), who emphasize the liquidity and safety services provided by short-term government securities. The demand for liquid assets is also affected by a time-varying, exogenous premium, η_t, which as discussed in Fisher (2015), can be interpreted as a change in the demand for risk-free bonds relative to risky assets. The operator, $\mathbb{E}\left[\cdot|\Omega_0\right]$ corresponds to the expectation conditional on information that the household has at time 0. The information set, Ω_t, includes the history of all the model variables through time t but not, as discussed above, the current and past values of j_t and ϵ_{Rt}.

The household receives labor income $\frac{W_t}{P_t}H_t$, where P_t and W_t denote the aggregate (consumption) price level and the nominal wage, respectively. Households also trade in financial markets where they can purchase or sell a one period risk-free nominal bond, B_{t+1}, which has a return between period t and period $t+1$ of R_t. In addition, households trade shares, E_{t+1}, in the economy's firms at price, P_t^E, and a share entitles a household to a dividend payment from the firms, D_t. The household maximizes equation (5) subject to the sequence of budget constraints,

$$C_t+\frac{B_{t+1}+P_t^E E_{t+1}}{P_t}=\frac{W_t}{P_t}H_t+\frac{R_{t-1}B_t}{P_t}+\frac{(P_t^E+D_{t-1})E_t}{P_t}-T_t, \tag{6}$$

where T_t are lump sum taxes (or transfers, if negative) from the government.

A household's optimal choices for bonds and equity imply:

$$\lambda_t=\beta R_t\mathbb{E}\left[\lambda_{t+1}\pi_{t+1}^{-1}|\Omega_t\right]+\frac{\eta_t P_t}{B_{t+1}}, \tag{7}$$

$$\mathbb{E}\left\{\beta\frac{\lambda_{t+1}}{\lambda_t}\pi_{t+1}^{-1}\left(R_{t+1}^E-R_t\right)|\Omega_t\right\}=\frac{\eta_t P_t}{\lambda_t B_{t+1}}, \tag{8}$$

where λ_t is the Lagrange multiplier on a household's budget constraint satisfying $\lambda_t = \frac{1}{C_t - \gamma C_{t-1}^a}$, π_{t+1} is the inflation rate defined as $\pi_{t+1} = \frac{P_{t+1}}{P_t}$, and R_{t+1}^E is the return on equity satisfying $R_{t+1}^E = \frac{P_{t+1}^E + D_t}{P_t^E}$. The term, $\frac{\eta_t P_t}{B_{t+1}}$, in equation (7) reflects the non-pecuniary benefits of holding the risk-free bond. Holding all else equal, equation (8) indicates that an increase in the demand for the risk-free bond (i.e., a higher value of η_t) leads to an increase in the risk premium, while an increase in its net supply will reduce it. The preference for the risk-free bond introduces discounting into the intertemporal Euler equation, mitigating the effects on aggregate demand of future policy rate changes (as discussed in Fisher (2015) and Campbell, Fisher, Justiniano, and Melosi (2016)). A household supplies its labor services in a perfectly competitive market and their labor supply satisfies $\frac{W_t}{P_t}\lambda_t = \chi_0$.

3.3 Firms

There is a continuum of monopolistically competitive firms producing differentiated intermediate goods. These goods are used as inputs by a perfectly competitive firm producing a single final good. All the firms in the economy make their decisions using the same information set, Ω_t, and thus do not observe the current monetary policy regime.

The final good is produced with a constant returns technology, $Y_t = \left(\int_0^1 Y_t(f)^{\frac{\epsilon-1}{\epsilon}}\, df\right)^{\frac{\epsilon}{\epsilon-1}}$, where $Y_t(f)$ is the quantity of intermediate good f used as an input, and $\epsilon > 1$ is its elasticity of demand. Profit maximization, taking as given the final goods price P_t and the prices for the intermediate goods $P_t(f)$, for all $f \in [0,1]$, yields the following set of demand schedules,

$$Y_t(f) = \left(\frac{P_t(f)}{P_t}\right)^{-\epsilon} Y_t. \tag{9}$$

The zero profit condition implies $P_t = \left(\int_0^1 P_t(f)^{1-\epsilon}\, df\right)^{\frac{1}{1-\epsilon}}$.

An intermediate goods firm combines capital, $K_t(f)$, and labor, $H_t(f)$ to produce its good according to:

$$Y_t(f) = Z_t K_t(f)^{\alpha} H_t(f)^{1-\alpha}, \tag{10}$$

where $0 < \alpha < 1$ and Z_t is the aggregate level of technology. Capital is in fixed supply in the model though intermediate goods producers can buy and sell capital from each other at the price q_t in a perfectly competitive market. The firm also hires the labor services of the households in a perfectly competitive market at the wage $\frac{W_t}{P_t}$.

Intermediate goods firm f sells its differentiated good in a monopolistically competitive market and can choose its nominal price $P_t(f)$ subject to the requirement that it satisfies the demand of the representative final goods producer at that price. Following Rotemberg (1982), the intermediate good producer faces a quadratic cost of adjusting its nominal price between periods, which is measured in terms of the final good:

$$\varphi_t^p(f) = \frac{\varphi}{2}\left[\frac{P_t(f)}{\pi_{t-1}^{1-a}\bar{\pi}^a P_{t-1}(f)} - 1\right]^2 Y_t.$$

The parameter $\varphi \geq 0$ governs the size of the adjustment cost and the indexation parameter a satisfies $0 \leq a \leq 1$ and allows for the adjustment costs to depend on a weighted average of lagged inflation and $\bar{\pi}$, the target inflation rate set by the central bank.

Firm f chooses state contingent plans for capital, hours, and its price to maximize the present discounted value of expected profits:

$$\mathbb{E}\left\{\sum_{t=0}^{\infty}\beta^t\lambda_t\left[(1+\tau)\left(\frac{P_t(f)}{P_t}\right)Y_t(f) - w_t H_t(f) - q_t(K_{t+1}(f) - K_t(f)) - \varphi_t^p(f)\right]|\Omega_0\right\}, \tag{11}$$

subject to equations (9) and (10). In equation (11), τ is a subsidy to firms that is financed out of lump-sum taxes and q_t is the price of an additional unit of capital.

In symmetric equilibrium all intermediate goods producers make the same decisions: $K_{t+1}(f) = K_{t+1}$, $H_t(f) = H_t$, and $P_t(f) = P_t$. In this case, the firm's pricing equation can be written:

$$f_\pi\left(\pi_t, \pi_{t-1}\right) = \beta\mathbb{E}\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{Y_{t+1}}{Y_t}f_\pi\left(\pi_{t+1}, \pi_t\right)|\Omega_t\right] + \frac{\epsilon}{\varphi}\left[mc_t - \frac{\epsilon - 1}{\epsilon}(1+\tau)\right], \tag{12}$$

where the function, $f_\pi\left(\pi_t, \pi_{t-1}\right)$, is given by

$$f_\pi\left(\pi_t, \pi_{t-1}\right) = \varphi\left[\frac{\pi_t}{\pi_{t-1}^{1-a}\bar{\pi}^a} - 1\right]\frac{\pi_t}{\pi_{t-1}^{1-a}\bar{\pi}^a}.$$

A firm's marginal cost is given by $mc_t = \frac{W_t H_t}{(1-\alpha)P_t Y_t}$. Arbitrage between the capital and financial markets implies that the return on equity is equal to the return on capital so that $R_{t+1}^E = \frac{q_{t+1}}{q_t} + \alpha mc_t \frac{Y_{t+1}}{q_t K_{t+1}}$.

3.4 Shocks

In addition to the unobserved monetary regimes and the monetary policy shocks, j_t and e_{Rt}, the economy fluctuates in response to shocks to technology and the 'risk premium'. Both of these shocks are observed by households and firms. The technology shock evolves according to:

$$\log(g_{Zt}) = \rho_Z \log(g_{Zt-1}) + \sigma_Z e_{Zt}, \tag{13}$$

where $g_{Zt} = \log(Z_t) - \log(Z_{t-1})$, $0 \leq \rho_\eta < 1$, $\sigma_Z > 0$, and e_{Zt} is a normally-distributed innovation with zero mean and a standard deviation of unity.

The stochastic process for the risk premium shock, η_t is given by:

$$\eta_t = (1 - \rho_\eta)\eta + \rho_\eta \eta_{t-1} + \sigma_\eta e_{\eta_t}, \tag{14}$$

where η is the non-stochastic steady state value of η_t, $0 \leq \rho_\eta < 1$, $\sigma_\eta > 0$, and $e_{\eta t}$ is a normally-distributed innovation with zero mean and a standard deviation of unity. .

3.5 Monetary Policy

The reaction function for the central bank is given by:

$$f_R\left(\mathbb{X}_t, \Gamma_{j_t}\right) = \rho_R \log R_{t-1} + (1 - \rho_R)\left[\log R + \gamma_\pi \log\left(\frac{\pi_t}{\bar{\pi}}\right) + \gamma_y \log(\tilde{y}_t)\right] + \Gamma_{j_t}, \tag{15}$$

where $\tilde{y}_t$ denotes the output gap, $\mathbb{X}_t = \left\{R_{t-1}, \tilde{y}_t, \frac{\pi_t}{\bar{\pi}}\right\}$, and $R = \frac{(1-\eta)\bar{\pi}}{\beta}$. In equation (15), $\tilde{y}_t$ is the output gap defined using the detrended level of output, $\tilde{y}_t = \frac{Y_t}{Z_t}$. We use this definition of the output gap rather than one that involves the level of output in the flexible price economy, because this concept corresponds more closely to the CBO's measure of the output gap, which we use to estimate the model. As shown in the appendix, using $\tilde{y}_t$ for the output gap is equivalent to using the deviation of detrended output from its steady state value since the steady state value of $\tilde{y}_t$ is equal to one.

We model regime changes as affecting the intercept of an interest-rate rule through the term Γ_{j_t}. As discussed in the Introduction, this approach aligns well with the forward guidance communications by the Federal Reserve in 2014-15. Moreover, in our view the nature of the Federal Reserve's FG communications changed over time and an approach that imposes less structure is attractive given the evolving nature of FG communications.

In estimating the model, there are three regimes so that $N_j = 3$. We label and normalize the regimes *a priori* so that $\Gamma_{j_t} \in \{0, -\gamma_0, -2\gamma_0\}$ with $\gamma_0 > 0$. Our focus on regimes that imply a lower desired policy rate than would be the case under the systematic part of the rule reflects our interest in determining whether there was a regime change in monetary policy at a time when the Federal Reserve was attempting to provide additional accommodation through its FG communications.

We interpret the first regime as the "Normal Times" regime, and in this regime $f_R\left(\mathbb{X}_t, \Gamma_{j_t}\right)$ reflects only the systematic part of the rule. The second regime is called the "Easing" regime, as this regime is associated with a desired interest rate that is lower than in the "Normal Times" regime. As discussed further below, this regime helps capture the empirical regularity that the reductions in the federal funds rate during easing cycles occur more quickly than interest-rate increases during tightening cycles. The third regime is called the "FG" regime, and in this regime the desired policy rate can be substantially lower the it would be in the other two regimes.

The three regimes also differ in terms of their likelihood and persistence, as specified by the transition matrix, $\mathbf{P}$. We discuss in detail our prior for $\mathbf{P}$ in the next section but the priors can be summarized as follows. The priors imply the Normal Times regime is persistent with most of the realizations of j_t associated with this regime. The FG regime is also specified to be persistent though not as persistent as the Normal Times regime. Moreover, the priors imply that the likelihood of being in the FG regime is low relative to the Normal Times regime. The likelihood of being in the Easing regime is also specified to be low and our priors impose that this regime is short-lived relative to the other two regimes.

As discussed above, agents have incomplete information about the regimes since they do not observe j_t and e_{Rt}. Because of this assumption, the regimes are imperfectly credible since the actual realization of the regime can differ from what agents believe. To understand the quantitative importance of this assumption, we contrast this specification with the case in which agents have complete information so that they observe both j_t and e_{Rt}. In that case, regime changes are fully credible.

3.6 Market Clearing

The government budget constraint is:

$$\frac{B_{t+1}}{P_t} + T_t = \frac{R_{t-1}B_t}{P_t} + \tau Y_t. \tag{16}$$

The market clearing conditions for the final goods is:

$$Y_t = C_t + \frac{\varphi}{2}\left(\frac{\pi_t}{\pi_{t-1}^{1-a}\bar{\pi}^a} - 1\right)^2 Y_t, \tag{17}$$

and the supply of capital and the number of equity shares is normalized to one (i.e., $K_t = E_t = 1$).

4 Solution and Econometric Inference

Appendix B shows the model's equilibrium conditions. The model is solved for a minimum state variable solution using a projection method similar to Gust, Herbst, López-Salido, and Smith (2017).[4] We prefer such an approach because of the nonlinearities coming from the ZLB and the way in which agents update their beliefs. Because we are estimating the model, it might be tempting to use a computationally-efficient solution algorithm that respects the nonlinearity in the Taylor rule but log-linearizes the remaining equilibrium conditions. However, as discussed in Gust, Herbst, López-Salido, and Smith (2017), this approach can perform poorly quantitatively and lead to large differences in some of the estimated parameters.

4.1 Data and Estimation

Data. We use quarterly data on the output gap, inflation, and the interest rate for the United States from 1992:Q1 - 2016:Q4. The output gap is constructed as the log difference between actual output and the Congressional Budget Office's (CBO) estimate of potential GDP. Using this measure of real activity ensures that the model captures the protracted downturn following the financial crisis. Inflation is measured as the percent change in core PCE prices which excludes changes in energy and food prices. For the policy rate, we use the quarterly average of the federal funds rate. The time series for the data are plotted in Figure 1.

Estimation. The solution algorithm described above characterizes the transition for the model's state variables s_t, as a function of its past realization, s_{t-1}, and current innovations to the shocks, ε_t.[5] The minimum set of state variables for the model includes

$$s_t = [R_t, \pi_t, Y_t, C_t, g_{Zt}, \eta_t, j_t]\,. \tag{18}$$

The state variables are updated according to

$$s_t = \Phi(s_{t-1}, \varepsilon_t; \theta), \quad \varepsilon_t \sim N(0, I), \tag{19}$$

[4] The minimum state variable solution rules out equilibria with additional state variables such as a sunspot or those with nonstationary dynamics. See McCallum (1999) for a discussion. However, Benhabib, Schmitt-Grohe, and Uribe (2001) emphasize that there is a second steady state that is deflationary due to the imposition of the ZLB on the Taylor rule. Our solution algorithm in principle does not rule out dynamics that fluctuate around this deflationary steady state. But, in practice, we find that that the solution algorithm does not converge to this unintended deflationary equilibrium.

[5] Note that j_t is part of the state vector. For the version of the model with incomplete information, j_t does not enter the agents' decision rules. However, it remains part of the overall transition of the state even in that version of the model.

where Φ denotes the transition function, ε_t is a vector containing the three innovations to the shocks as well as an innovation which we use to govern the Markov-switching process for j_t, and θ is a vector containing the model's parameters.[6]

The data are linked to the model via the observation equations:

$$\text{Output Gap}_t = \log(\tilde{y}_t) + v_{y,t}, \tag{20}$$

$$\text{Inflation}_t = 4\log(\pi_t) + v_{\pi,t}, \tag{21}$$

$$\text{Federal Funds Rate}_t = 4\log(R_t) + v_{r,t}. \tag{22}$$

According to these equations, each of the series is subject to a small amount of measurement error. The standard deviation of the measurement error is set equal to 1 percent of the standard deviation of each of the series over the estimation period.

Taken together, equations (19), (20), (21), and (22) form a nonlinear state space model. We use this econometric model to estimate the parameters of the model, θ, to assess the likelihood of changes in monetary policy regimes and to assess the fit of the incomplete information version of the model relative to the full information version. To do so, we take a Bayesian approach, similar to many papers in the literature that estimate DSGE models, in which the parameters, θ, are random variables whose initial distribution (the "prior") is updated in light of the observed data to a new distribution (the "posterior").

We use simulation techniques to elicit random draws from the posterior distribution. Specifically, we use the Random Walk Metropolis-Hastings (RWMH) algorithm to generate draws from a Markov chain $\{\theta^i\}_{i=1}^N$ which converge to the posterior distribution. A well known complication when using nonlinear state space models in this setting is that the likelihood function, $p(Y|\theta)$, is not available in closed form. Instead, we rely on sequential Monte Carlo techniques known as particle filters to estimate the likelihood function at each step of the RWMH algorithm. Herbst and Schorfheide (2015) provide an overview of both the RWMH algorithm and particle filtering nonlinear state space models.

Particle filtering for DSGE models is challenging because of the both the size of state space and because of model misspecification. Typically, to obtain reliable likelihood estimates, a large number of particles are necessary and/or a relatively high degree of measurement error. In our application, we seek to use a relatively small amount of measurement error so that we can focus on whether there was

[6] In our implementation, this innovation is a standard normal random variable that is mapped into the appropriate outcome space for the Markov-switching process using an inverse CDF.

a policy regime change at the zero lower bound. To avoid having to use an infeasibly high number of particles, we use the adaptive Tempered Particle Filter proposed by Herbst and Schorfheide (2018). This filter augments the system with artificially high measurement error which is adaptively reduced to the desired amount. As discussed in Herbst and Schorfheide (2018), it allows for reliable likelihood estimates with moderate amounts of particles, even for systems with small amounts of measurement error.[7]

Prior. Before detailing the prior distribution for the estimated parameters, we note that we fix a subset of parameters. In particular, we fix the parameters that are not directly related to the policy rule, regime switching process, or shock processes. This helps sharpen inference somewhat on the remaining parameters, but strictly speaking is not necessary. The results are robust to estimating the parameters we fix under reasonable prior distributions. The parameters we fix are η, γ, a_p, $\bar{\pi}$, and φ. The parameter η governs the size of the risk premium and we set $\eta = 0.25\%$ so that it implies an annualized risk premium of 1%. The habit parameter, γ, and the price indexation parameter, $1 - a_p$, are set to 0.70 and 0.15, respectively. The central bank's target for inflation, $\bar{\pi}$, corresponds to 2%, in line with FOMC's stated objective and close to the average rate of inflation over our sample period. The Rotemberg parameter (φ) controlling the cost of price changes is set to 100, which can be shown to be consistent with a frequency of price changes of slightly more than one year in a model with Calvo contracts.

The prior distribution for the estimated parameters are displayed in Table 1. The prior distribution for the interest rate rule is moderately informative, whose mean corresponds to a central bank which responds to inflation, but not the output gap, and which exhibits moderate persistence. The marginal prior distributions for the parameters associated with the risk premium and technology shock are less informative.

A key component of the prior distribution is the marginal prior for γ_0, which governs the values of Γ_{j_t}, $j_t \in \{1, 2, 3\}$. We set the prior for γ_0 to be identical to the prior for σ_r. That is, our prior views indicate that the Easing regime represents a (negative) one standard deviation monetary policy shock, and the FG regime represents a two standard deviation monetary policy shock.

The parameters governing the transition matrix, $\mathbf{P}$, are also key components of the prior distribution. We set the prior for $\mathbf{P}$ to be consistent with the regime descriptions discussed earlier. Specifically,

[7] Details about the settings and the stability of the Tempered Particle Filter are available upon request.

we parameterize the $\mathbf{P}$ matrix using four parameters, $[P_{11}, P_{33}, p_\alpha, p_\beta]$ as:

$$\mathbf{P} = \begin{pmatrix} P_{11} & 1 - P_{11} & 0 \\ p_\beta(1 - p_\alpha P_{33}) & p_\alpha P_{33} & (1 - p_\beta)(1 - p_\alpha P_{33}) \\ (1 - P_{33})/2 & (1 - P_{33})/2 & P_{33} \end{pmatrix} \tag{23}$$

where the element p_{ij} represents the $\mathbb{P}\{j_t = j | j_{t-1} = i\}$. Our prior beliefs can be summarized as follows. The Normal Times regime is the dominant regime, with most of the j_t realizations associated with this regime. Moreover, the regime is quite persistent. Finally, it is not possible to transit from the Normal Times regime to the FG regime without first visiting the Easing regime.

We operationalize these beliefs by fixing $P_{13} = 0$ and thus $P_{12} = 1 - P_{11}$. The prior on P_{11} is a Beta distribution with mean 0.95 and standard deviation 0.03. This prior corresponds to a distribution for the expected duration of remaining in the Normal Times with a mean of 20 quarters and 10 and 100 quarters, respectively, at the 5th and 95th percentiles. The prior on P_{33}, which determines the duration of the FG regime, is given by a Beta distribution with mean 0.90 and standard deviation of 0.05. This prior corresponds to a distribution for the expected duration of remaining in the FG regime with a mean of 10 quarters and 8 and 33 quarters, respectively, at the 5th and 95th percentiles. The priors for the transition matrix also imply that it is equally likely that j_t moves from the FG regime to the Normal Times or Easing regimes, as the probabilities for P_{31} and P_{32} are both parameterized to be $(1 - P_{33})/2$.

The transition probabilities conditional on being in the Easing regime are indirectly parameterized via p_α and p_β. The Easing regime is meant to capture quick, easing cycles in which the federal funds rate is sharply cut. To accomplish this, we use priors that imply the expected duration of the Easing regime is strictly less than that of the FG regime by setting $P_{22} = p_\alpha P_{33}$, where the prior distribution for p_α is a Beta distribution with a mean of 0.50 and a standard deviation of 0.10. The priors for P_{21} and P_{23} are chosen so that transitioning from the Easing regime to the FG regime is substantially less likely than transitioning from the Easing regime to the Normal Times regime. To do so, $P_{21} = p_\beta P_{22}$ and $P_{32} = (1 - p_\beta)P_{22}$ where p_β has a Beta prior distribution with a mean of 0.90 and a standard deviation of 0.05.

In summary, the prior distributions for the four parameters, $[\gamma_0, P_{11}, P_{33}, p_\alpha, p_\beta]$, govern the regime switching process and are key components of the estimation strategy. Table 2 displays the implied prior means and marginal 5th and 95th percentiles of the elements of γ_0, $\mathbf{P}$, and the ergodic distribution for the regime switching process. As indicated by the ergodic probabilities, the priors imply that the likelihoods of the Normal Times regime, the Easing regime, and the FG regime are 87 %, 8 %, and 5%, respectively. Our approach imposes quite a bit of structure about the regimes in order to help distinguish them from each other. This is often necessary in structural models with Markov switching such as ours, because the model is highly nonlinear in its parameters. Having said that, we also estimated the model using the methodology in Rouwenhurst to characterize the transition matrix, $\mathbf{P}$, and found that the results were broadly similar.

5 Estimation Results

Parameter Estimates. Table 3 displays the posterior mean and marginal 5th and 95th percentiles of the posterior distribution of the estimated parameters for three different specifications of the model: the version with no regime changes, the FI version, and the II version. With the exception of the estimate for β and the parameters governing the policy regimes, the parameter estimates are similar across the three specifications. The estimates for the policy rule indicate that monetary policy responds aggressively to both inflation and the output gap with the inflation response slightly greater than in Taylor (1993) and the response to the output gap substantially greater than in Taylor (1993).[8] With the posterior mean of ρ_r above 0.8, our results also suggest considerable inertia in the setting of the federal funds rate. In all three versions of the model, the estimated volatility of the monetary policy innovations are small relative to those of the risk premium and markup shocks.

The estimate of β is notably lower in the versions of the model with regime switching than in the version with no changes in monetary policy regimes. This lower value of β has the effect of raising the average nominal rate in these regime-switching versions of the model. Still, the average nominal rate in the three versions of the model is similar because in the two versions with regime switching the fact that γ_0 is positive on average puts downward pressure on the average nominal rate.

Table 3 shows that in the FI specification of the model a change in regime has about the same contemporaneous impact as a one standard deviation innovation in the policy rule, as the mean estimate of γ_0 is only slightly larger than that of σ_r. However, in the II version of the model, a regime change is larger, as the mean estimate of γ_0 is about 75% larger than a one standard deviation innovation in

[8] We use data expressed at a quarterly rate to estimate the model. Hence, a coefficient of 0.4 on the output gap corresponds to a coefficient of 1.6 if the policy rule is expressed in annual terms as in Taylor (1993).

monetary policy. This larger estimate for γ_0 reflects that regime changes in the II specification are imperfectly credible and thus an equal-sized change in regime in this specification, all else equal, has a smaller effect on inflation and economic activity than in the FI specification. Hence, the estimated regimes are more distinct in the II specification than the FI specification.

Table 3 also shows the estimates of the parameters governing the transition matrix for the regimes, $\mathbf{P}$. In both the FI and II specifications, the Normal Times regime has an average duration of about 16-20 quarters. The FG regime is noticeably more persistent in the II version of the model than the FI version of the model. In particular, the FG regime has an average duration of about 6 quarters under FI and about 11 quarters under II. Accordingly, our estimates imply that, conditional on being in the FG regime, spells at the ZLB are likely to be longer in the II specification of the model than the FI specification.

Model Fit. Table 4 shows estimates of the log marginal data densities for the three versions of the model for the full sample and for the pre-crisis sample. The data favors the regime-switching versions of the model relative to the version with no switches in regimes. Interestingly, there is little difference in the marginal data densities between the FI and II versions of the model using the sample period before the ZLB spell, but the II version fits noticeably better for the full sample. Thus, this better fit mainly reflects the II specification's better performance over the ZLB episode. The better performance of II version of the model stands in contrast to the previous work of Schorfheide (2005), who estimates a similar model incorporating incomplete information but finds that the FI specification fits the data better than the II specification. This difference in results in part reflects that Schorfheide (2005) did not incorporate ZLB into his analysis and in particular how it affects the way in which agents learn about regimes, as that paper used an earlier sample period. Overall, the estimates suggest that incorporating regime changes and incomplete information during the period in which the federal funds rate was at the ZLB is important in explaining movements in interest rates, inflation, and the output gap during this period.

To understand why the II specification fits better than the FI model, Figure 2 displays the estimated innovations in the technology shock in the II and FI specifications of the model. Because these innovations are quite volatile, Figure 2 displays a moving-average representation of the innovations using eight lags. As shown in the upper panel of the figure, the estimated technology innovations in the II and FI specifications are very similar prior to the financial crisis. However, as shown in the lower panel, the innovations are noticeably larger in the FI specification than in the II specification during the ZLB episode. Because these innovations are assumed to be standard normal random variables, the marginal data density penalizes the FI specification of the model more than it does the II specification; hence, the II specification has a higher marginal data density than the FI specification.

The relatively large innovations of the FI specification are a manifestation of the forward guidance puzzle present in that specification of the model. Because agents view switches to either the asymmetric or FG regimes with perfect certainty in that version of the model, a change to either one of those regimes can have a large stimulative effect on inflation. With those regimes playing an important role during the 2008-2015 period, relatively large and positive innovations in the technology shock are necessary to offset the stimulative effect on inflation and keep it near observed inflation rate. While the II version also features positive innovations in technology over this period, they are not as large, because the change to the FG regime is viewed by agents as imperfectly credible and thus has less impact on inflation.

5.1 Estimated Policy Regimes

Filtered Probabilities. The solid lines in Figure 3 display the posterior mean estimates of filtered probabilities of the three regimes derived from the II version of the model. These probabilities are those that private-sector agents in the model assign to the three regimes. Because of the small degree of measurement error, the filtered or real time probabilities that an econometrician would assign to the regimes are quite similar. Interpreting these probabilities through the lens of the econometrician, they indicate that there have been changes in monetary policy regimes with the bulk of the evidence in favor of regime changes coming from the period during and after the financial crisis in 2007-2008. Prior to that crisis, the upper panel shows that the Normal Times regime was the dominant regime though there is some evidence of regime change over this period. In particular, the probability of the Normal Times regime falls below 50% during the 2001 recession in which the federal funds rate was cut sharply; hence, there is significant probability that the policy was governed by the Easing regime in 2001 and 2002. However, overall, the estimates suggest that prior to the financial crisis monetary policy could be described reasonably well by a rule with constant coefficients – a result that is consistent with those of Sims and Zha (2006).

Both the Easing and FG regimes play an important role at the onset and in the aftermath of the financial crisis. As the federal funds rate fell from around 5% to 50 basis points in late 2007 and 2008, the likelihood of being in the Normal Times regime falls sharply. This decline in the probability of the Normal Times regime at first corresponds to a large increase in the probability of being in the Easing regime in late 2007. With the federal funds rate continuing to fall sharply in 2008, the probability of being in the Easing regime falls and the probability of being in the more accommodative FG regime rises to above 90%. However, from the perspective of the agents in the model, the FG regime ended surprisingly quickly in 2008. This abrupt, unanticipated ending reflects that there was a pause in interest-rate cuts during 2008 that left the federal funds rate above the ZLB despite a large fall in the output gap. As a result, in early 2009 when the federal funds rate first reaches the ZLB, private sector agents largely believed that monetary policy was reacting to developments consistent with their behavior prior to the crisis. In particular, in 2009 they assigned more than a 70% chance to being in the Normal Times regime. Gradually agents' beliefs begin to change, as the probability of being in the FG regime rises over the 2009-2012 period. During 2012, the probability of being in the FG regime picks up noticeably and continues to rise in 2013 and 2014 and by the end of 2015 the probability of being in the FG regime is close to one. Subsequently in 2016 when the federal funds rate departs from the ZLB, there is a sharp fall in the likelihood of being in the FG regime and a concomitant increase in the likelihood of being in the Normal Times regime.

The dashed lines in Figure 3 display the estimates of the posterior mean filtered probabilities for the three regimes derived from the FI specification of the model. In this case, these probabilities correspond to the real time estimates from the perspective of the econometrician, as agents in the FI specification observe the policy regime. The estimates of the filtered probabilities from the FI model are broadly similar to the II model. In particular, the Normal Times regime is the dominant regime prior to the financial crisis and there is a temporary switch to the FG regime in 2008. However, the FG regime does not return with appreciable probability until 2014 – a couple of years later than in the II model. As discussed above, the II version of the model features sharper distinctions between the regimes and fits better than the FI version and thus we emphasize the results from that specification.

Smoothed Probabilities. Figure 4 compares the probabilities that agents in the II specification assign to the regimes in real time (solid line) over the 2009-2017 period to the posterior mean smoothed probabilities that the econometrician assigns to the regimes (dashed line). The latter probabilities take into account that the econometrician has the benefit of using hindsight to estimate the regimes using the data over the entire sample period. As discussed above, the real-time estimates suggest that when the federal funds rate reached the ZLB in early 2009, the probability that agents assigned to the FG regime was low, less than 15%. The agent's filtered probabilities do not rise above 50% until 2012 and do not go over 90% until 2015. By contrast, the smoothed probabilities indicated that an econometrician, acting with the benefit of hindsight, would be able to identify the FG regime much earlier. In particular, the smoothed probability for the FG regime is above 50% in early 2011 and is above 90% in 2012. Accordingly, our estimates suggest that it was difficult in real time for agents to learn about the switch to the FG regime.

The ZLB constraint plays a key role in making it difficult for agents to learn about the switch to the FG regime. In that case, agents no longer observe the desired policy rate and thus the likelihood function (shown in equation 3) changes to reflect that they have less information about the regimes. Figure 5 displays the regime probabilities in the unconstrained version of the model in which the ZLB constraint is not imposed.[9] In that case, there is little difference between the smoothed (dashed lines) and the filtered probabilities (solid lines). In the unconstrained model, the switch to the FG regime is identified by the econometrician with the benefit of hindsight in 2009 and the agents are able to identify it very quickly as well. In the unconstrained version of the model, the desired policy rate is directly observed by agents and is equal to the actual policy rate so that

$$\log R_t = f_R\left(\mathbb{X}_t, \Gamma_{j_t}\right) + e_{R_t}.$$

In addition, the likelihood function is no longer a mixture of two distributions and is given by

$$\xi\left(R_t, \mathbb{X}_t | j_t = j\right) = \frac{1}{\sigma_R}\phi\left(\frac{\log R_t - f_R\left(\mathbb{X}_t, \Gamma_j\right)}{\sigma_R}\right). \tag{24}$$

Using equation (24), agents can score each regime using the normal pdf. Given the large differences in Γ_j estimated in the incomplete information version of the model, this scoring leads to sharp distinctions between regimes; thus, agents quickly infer the switch to the FG regime.

5.2 The Macroeconomic Effects of Forward Guidance

To understand the importance of the FG regime for the macroeconomy, the solid lines in Figure 6 display the paths of inflation, the output gap, and the federal funds rate for the II version of the model.

[9] In this figure, we use the estimated parameter values and states from the incomplete information model imposing the ZLB. We then uses those estimates to simulate data from the unconstrained version of the model.

The solid line in the bottom panel of the Figure 6 also shows the associated probability that private sector agents assign to being in the FG regime. The dashed lines in the figure show the counterfactual paths of inflation, the output gap, and the federal funds rate holding $\Gamma_{jt} = \Gamma_1 = 0$ throughout the 2008-2016 period. Accordingly, it shows the outcomes under the assumption that monetary policy had stayed in the Normal Times regimes rather than switch to the more accommodative FG regime, and the dashed lines in the lower panel of Figure 6 shows how agents' beliefs would have evolved had this been the case. The difference between the solid and dashed lines in Figure 6 quantifies the effect of the switch from the Normal Times regime to the FG regime on the output gap and inflation over the 2009-2016 period and thus allows us to provide estimates of the effectiveness of the Federal Reserve's FG.

Figure 6 shows that FG had very little effect on economic outcomes over the 2009-2011 period. This is not surprising because, as shown in lower panel of Figure 6, agents put very little weight on the possibility of policy being in the FG regime through 2010. Their beliefs about being in the FG regime only begins to increase over the course of 2011 so that by the end of 2011, there are noticeable effects on output and inflation relative to the counterfactual in which policy remains in the Normal Times regime. Specifically, the output gap is about 25 basis points higher on average in 2012 and 2013 than in the counterfactual in which $\Gamma_{jt} = 0$ and inflation is about 20 basis points higher. In later years, as agents become increasingly certain that policy is responding to economic developments according to the FG regime, the effects of the change in regime on economic activity and inflation become larger. In 2014 and 2015, the output gap is about 0.40 percentage point higher, on average, than in the counterfactual with no regime change and inflation is about 20 basis points higher.

Figure 6 shows that even in the counterfactual with $\Gamma_{jt} = 0$ the federal funds rate remains at the ZLB from 2009-2015. All else equal, the fact that $\Gamma_{jt} = 0$ in the Normal Times regime should induce a higher federal funds rate, making it more likely for the federal funds rate to rise above the ZLB. However, output is further below potential and inflation is lower in the counterfactual simulation, which has the counterbalancing effect of making it more likely for the federal funds rate to be at the ZLB.

Figures 6 answer the question of how much lower the output gap and inflation would have been had there been no regime change in monetary policy following the financial crisis in 2008. Figure 7 addresses a different but related question: how would economic outcomes differ had a fully-credible, FG regime been introduced in 2009Q1. In particular, it shows the effects on the output gap and inflation in a counterfactual in which the policy regime is fixed at its estimated value in the FG regime from 2009 onwards when agents have full information. It shows these effects relative to the situation in which agents have incomplete information about this switch.

If agents have full information, Figure 7 shows that there can be very large effects on inflation and the output gap from the change in regime. In particular, the output gap is about 1.75 percentage points higher, on average, over the 2009-2015 period if the regime change is fully credible rather than imperfectly credible. The quarterly inflation rate is about 75 basis point higher which translates into an inflation rate that would have been 3 percentage points higher. These large differences in outcomes are a manifestation of the FG puzzle in the FI specification of the model and highlight the importance of introducing imperfect credibility into the empirical analysis: the regime change is much less effective in stimulating the economy when agents view it as imperfectly credible. In particular, as shown in the lower right panel of Figure 7, it takes considerable time before the agents learn about the switch to the FG regime under incomplete information. In particular, it takes almost 3 years before the agents assign a probability of being in the FG regime above 50% and about 5 years before that probability rises above 90%, though with the benefit of hindsight they would have learned about it much faster.

To further understand the role of the ZLB, Figure 8 compares the effects of the shift to the FG regime in 2009Q1 in the II version of the model ignoring the ZLB and imposing the ZLB. When the ZLB is ignored, as emphasized earlier, agents learn very quickly (dashed line in lower right panel). Accordingly, there are large effects on output and inflation from the change in regime in the II version of the model in which the ZLB is ignored. In particular, the output gap is about 1.75 percentage points higher, on average, over the 2009-2015 period in the unconstrained version of the II model than in the constrained version of that model.

6 Conclusion

We estimated a New Keynesian model in which there are monetary policy regime changes and agents have incomplete information about the regimes. We provided evidence that there was a change in the policy regime at the time the federal funds rate was at the ZLB. We called this regime the FG regime and showed that because agents had incomplete information about monetary policy they were slow to learn about this change in regime. We were able to generate this slow evolution of beliefs while assuming agents have rational expectations because of the role of the ZLB in limiting the information they have about monetary policy. Our estimates suggest that imperfect credibility played a significant role in limiting the effectiveness of the Federal Reserve's forward guidance. Our empirical analysis also reinforced the importance of incorporating incomplete information into structural models as we find that the incomplete information version of the model fits the data better than the full information version.

References

Benhabib, J., S. Schmitt-Grohe, and M. Uribe (2001): "Monetary Policy and Multiple Equilibria," *American Economic Review*, 91, 167–186.

Campbell, J. R., J. D. Fisher, A. Justiniano, and L. Melosi (2016): "Forward Guidance and Macroeconomic Outcomes Since the Financial Crisis," NBER Macroeconomic Annual 2016.

Cogley, T. W., and T. J. Sargent (2005): "Anticipated Utility and Rational Expectations as Approximations of Bayesian Decision Making," Working Papers 523, University of California, Davis, Department of Economics.

Cole, S. (2015): "Learning and the effectiveness of central bank forward guidance," Mpra paper, University Library of Munich, Germany.

De Graeve, F., P. Ilbas, and R. Wouters (2014): "Forward Guidance and Long Term Interest Rates: Inspecting the Mechanism," Working paper series no. 292, Sveriges Riksbank.

Del Negro, M., M. P. Giannoni, and C. Patterson (2015): "The Forward Guidance Puzzle," Federal Reserve Bank of New York Staff Report 574.

Engen, E. M., T. Laubach, and D. L. Reifschneider (2015): "The Macroeconomic Effects of the Federal Reserve's Unconventional Monetary Policies," Finance and Economics Discussion Series 2015-5, Board of Governors of the Federal Reserve System (U.S.).

Erceg, C. J., and A. T. Levin (2003): "Imperfect Credibility and Inflation Persistence," *Journal of Monetary Economics*, 50, 915–944.

Fisher, J. D. (2015): "On the Structural Interpretation of the Smets-Wouters "Risk Premium" Shock," *Journal of Money, Credit and Banking*, 47, 511–516.

Gertler, M. (1982): "Imperfect Information and Wage Inertia in the Business Cycle," *Journal of Political Economy*, 90(5), 967–987.

Geweke, J. (1999): "Using Simulation Methods for Bayesian Econometric Models: Inference, Development, and Communication," *Econometric Reviews*, 18(1), 1–126.

Gust, C., E. Herbst, D. López-Salido, and M. E. Smith (2017): "The Empirical Implications of the Interest-Rate Lower Bound," American Economic Review, forthcoming.

Gust, C., D. López-Salido, and M. E. Smith (2012): "The Empirical Implications of the Interest-Rate Lower Bound," Finance and Economics Discussion Papers, Federal Reserve Board, 2012-83.

Herbst, E., and F. Schorfheide (2015): *Bayesian Estimation of DSGE Models*. Princeton University Press, Princeton.

——— (2018): "Tempered Particle Filtering," *Journal of Econometrics*, forthcoming.

Ireland, P. N. (2011): "A New Keynesian Perspective on the Great Recession," *Journal of Money, Credit, and Banking*, 43, 31–54.

Krishnamurthy, A., and A. Vissing-Jorgensen (2012): "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 120(2), 233–267.

Lucas, R. E. (1972): "Expectations and the Neutrality of Money," *Journal of Economic Theory*, 4, 103–124.

McCallum, B. T. (1999): "Role of the Minimal State Variable Criterion in Rational Expectations Models," *International Tax and Public Finance*, 6, 621–639.

McKay, A., E. Nakamura, and J. Steinsson (2016): "The Power of Forward Guidance Revisited," *American Economic Review*, 106(10), 3133–58.

Rotemberg, J. (1982): "Sticky Prices in the United States," *Journal of Political Economy*, 90, 1187–1211.

Schorfheide, F. (2005): "Learning and Monetary Policy Shifts," *Review of Economic Dynamics*, 8(2), 392–419.

Sims, C. A., and T. Zha (2006): "Were There Regime Switches in U.S. Monetary Policy?," *American Economic Review*, 96(1), 54–81.

Svensson, L. E., and N. Williams (2008): "Optimal Monetary Policy Under Uncertainty in DSGE Models: A Markov Jump-Linear-Quadratic Approach," Discussion paper, National Bureau of Economic Research.

Taylor, J. B. (1993): "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy*, 39, 195–214.

Table 1: PRIOR DISTRIBUTION

Name	Density	Prior Para (1)	Para (2)
Interest Rate Rule			
$100(1/\beta - 1)$	Gamma	0.500	0.200
ρ_r	Beta	0.600	0.200
ψ_π	Normal	1.500	0.200
ψ_x	Normal	0.000	0.250
$100\sigma_r$	Inv. Gamma	0.100	4.000
Process for j_t			
γ_0	Inv. Gamma	0.100	4.000
P_{11}	Beta	0.950	0.030
P_{33}	Beta	0.900	0.050
p_α	Beta	0.500	0.100
p_β	Beta	0.900	0.050
Other Exogenous Processes			
ρ_η	Beta	0.600	0.200
$100\sigma_\eta$	Inv. Gamma	0.100	4.000
ρ_z	Beta	0.600	0.200
$100\sigma_z$	Inv. Gamma	0.100	4.000

Notes: Para (1) and Para (2) correspond to the mean and standard deviation of the Beta, Gamma, and Normal distributions and to the upper and lower bounds of the support for the Uniform distribution. For the Inv. Gamma distribution, Para (1) and Para (2) refer to s and ν, where $p(\sigma|\nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$.

Table 2: Implied Prior Distribution for Monetary Policy Regimes (Γ_{j_t})

Parameter	Mean	[05, 95]
	μ	
Γ_1	0.00	[0.00, 0.00]
Γ_2	-0.11	[-0.21, -0.05]
Γ_3	-0.23	[-0.44, -0.12]
	Transition Matrix **P**	
P_{11}	0.95	[0.90, 0.99]
P_{12}	0.05	[0.01, 0.10]
P_{13}	0.00	[0.00, 0.00]
P_{21}	0.50	[0.35, 0.64]
P_{22}	0.45	[0.30, 0.60]
P_{23}	0.06	[0.02, 0.12]
P_{31}	0.05	[0.02, 0.10]
P_{32}	0.05	[0.02, 0.10]
P_{33}	0.90	[0.81, 0.97]
	Ergodic Probabilities	
p_1	0.87	[0.72, 0.97]
p_2	0.08	[0.02, 0.15]
p_3	0.05	[0.01, 0.15]

Notes: The table denotes the mean, 5th, and 95th percentile of the prior distribution for Γ_{j_t}, **P**, and p (the ergodic probabilities of Γ_{j_t}) implied by the prior distribution for **P**.

Table 3: Posterior Estimates

Specification:	No Switches		Full Information		Incomplete Information	
	Mean	[05, 95]	Mean	[05, 95]	Mean	[05, 95]
$100(1/\beta - 1)$	0.78	[0.70, 0.84]	1.06	[0.90, 1.18]	1.11	[0.90, 1.25]
$100\sigma_r$	0.10	[0.10, 0.11]	0.08	[0.07, 0.09]	0.08	[0.07, 0.09]
ρ_η	0.87	[0.83, 0.91]	0.89	[0.86, 0.92]	0.87	[0.81, 0.92]
$100\sigma_\eta$	0.29	[0.22, 0.37]	0.28	[0.19, 0.36]	0.35	[0.22, 0.48]
ρ_z	0.38	[0.34, 0.43]	0.36	[0.31, 0.44]	0.38	[0.29, 0.45]
$100\sigma_z$	0.95	[0.78, 1.04]	0.94	[0.81, 1.04]	0.88	[0.76, 1.00]
ρ_r	0.86	[0.84, 0.88]	0.83	[0.79, 0.87]	0.85	[0.82, 0.88]
ψ_π	1.77	[1.57, 1.99]	1.82	[1.62, 2.19]	1.78	[1.46, 2.12]
ψ_x	0.42	[0.39, 0.46]	0.41	[0.33, 0.48]	0.42	[0.28, 0.51]
γ_0			0.10	[0.06, 0.14]	0.14	[0.10, 0.17]
P_{11}			0.94	[0.91, 0.97]	0.95	[0.90, 0.98]
P_{33}			0.83	[0.74, 0.92]	0.91	[0.85, 0.96]
p_α			0.51	[0.45, 0.62]	0.46	[0.36, 0.52]
p_β			0.80	[0.70, 0.88]	0.78	[0.72, 0.83]

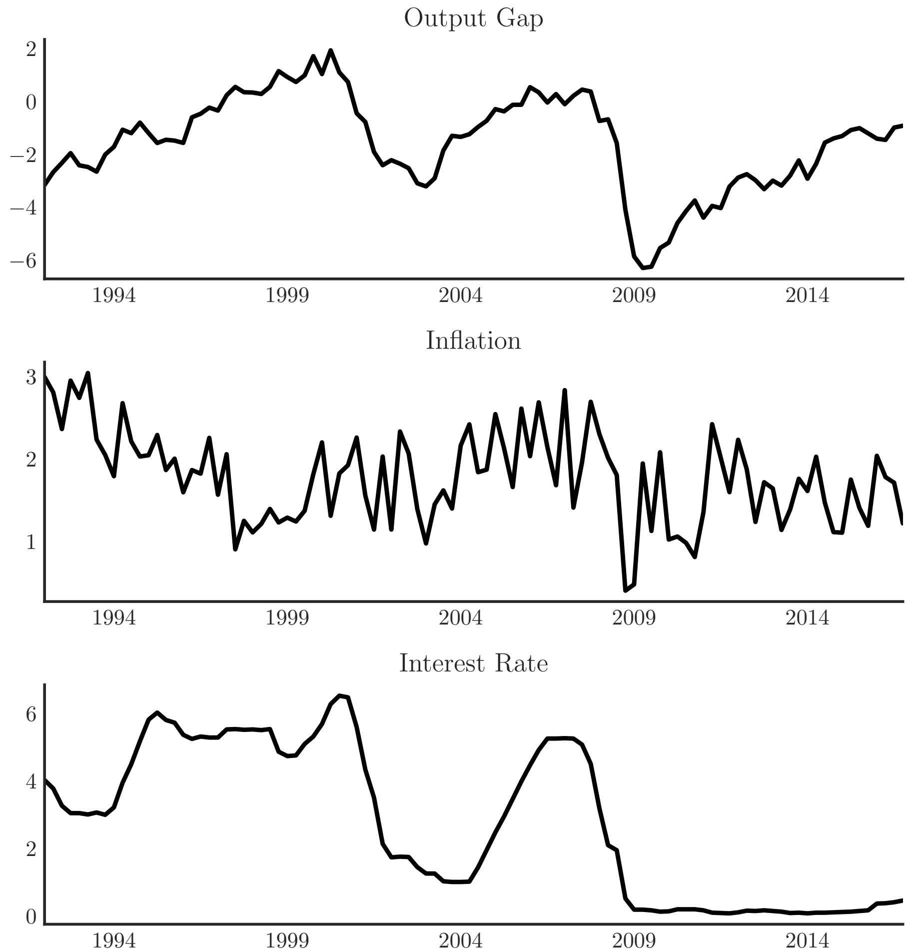
Notes: The table denotes the mean, 5th, and 95th percentile of the posterior distribution for the No Switches, Full Information, and Imperfect Information specifications of the model. The statistics are constructed using a MCMC chain of length $N = 40,000$ after a burn-in of $N_0 = 10,000$.

Table 4: Log Marginal Data Densities

Specification	Sample Period	
	1992-2016	1992-2007
No Switches	-187.03	-153.92
Full Information	-178.51	-144.92
Incomplete Information	-170.07	-145.50

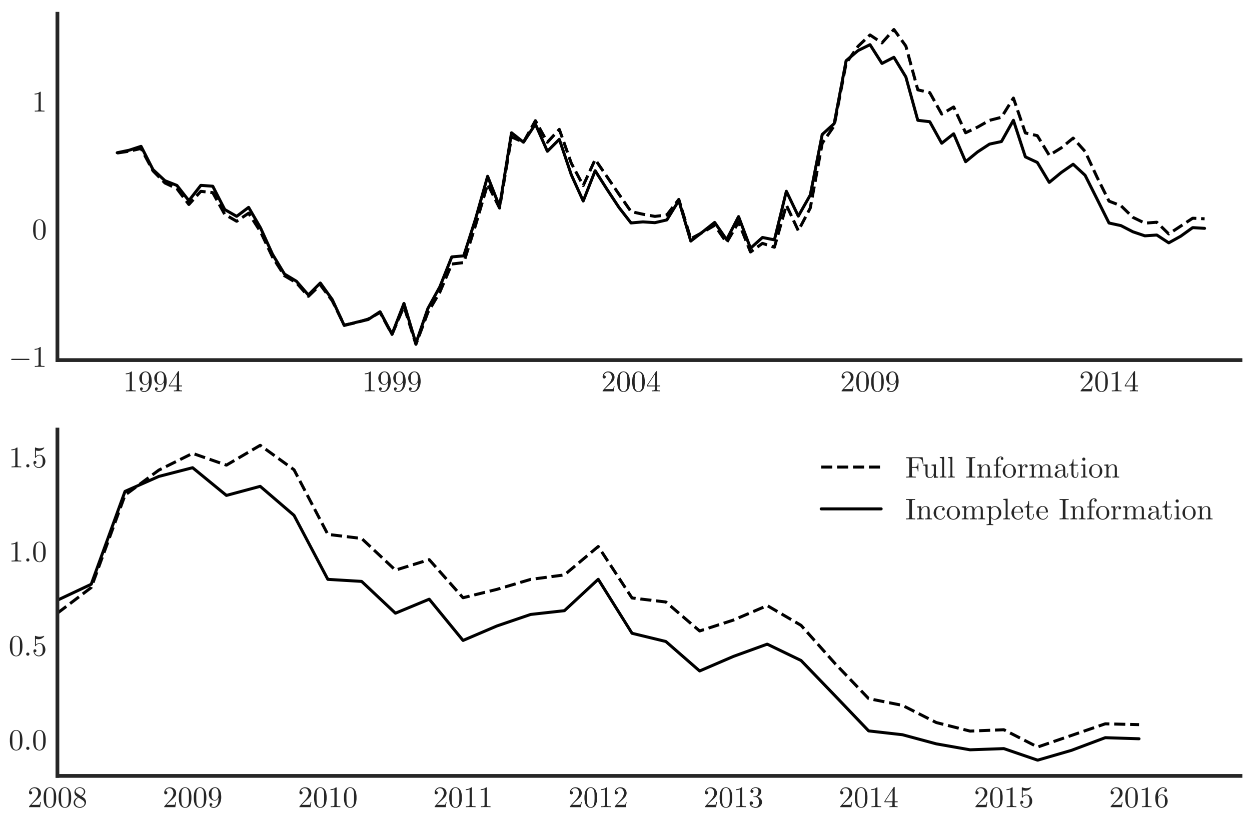
Notes: The table displays the estimates of log marginal data densities, $\log p(Y)$, a summary statistic of overall model fit. The estimates are computed using the output from the posterior simulator and the modified harmonic mean method of Geweke (1999).

Figure 1: Observables



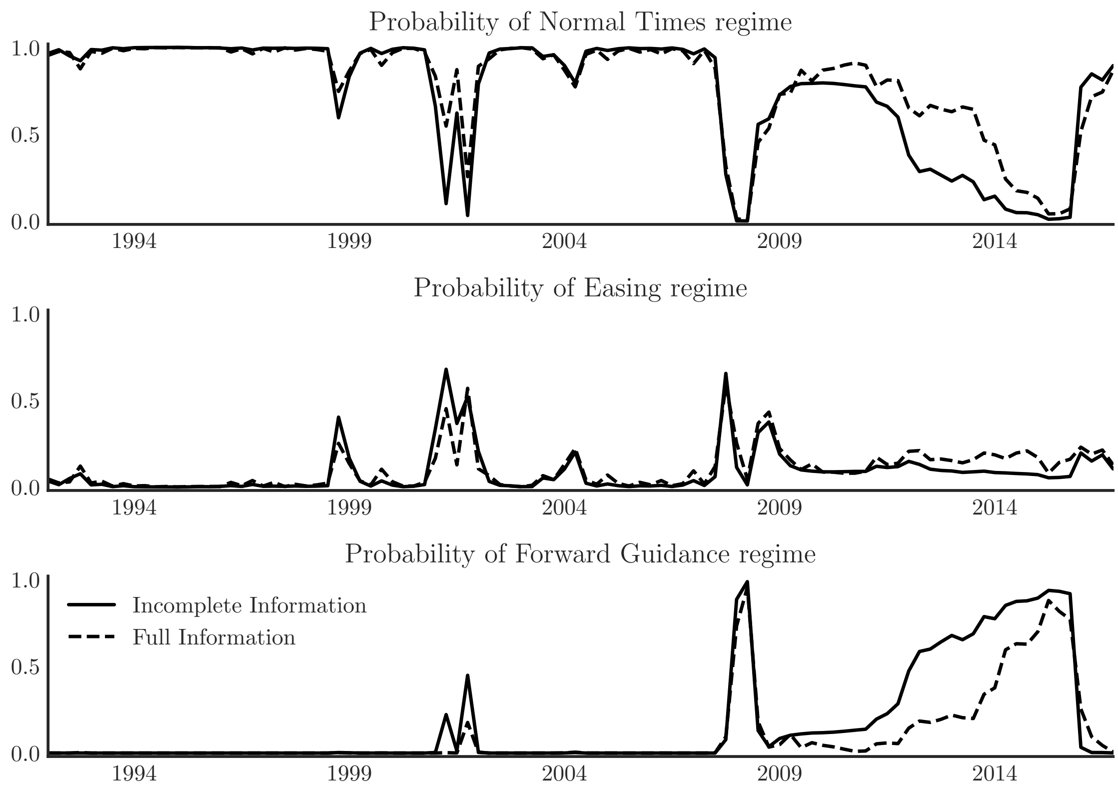
Notes: The figure displays the data we use to estimate the model. The output gap is constructed as the log difference between actual output and the Congressional Budget Office's (CBO) estimate of potential GDP. Inflation is measured as the percent change in core PCE prices, and for the policy rate the quarterly average of the federal funds rate is used.

Figure 2: Estimated Innovations in Technology: Incomplete and Full Information Specifications



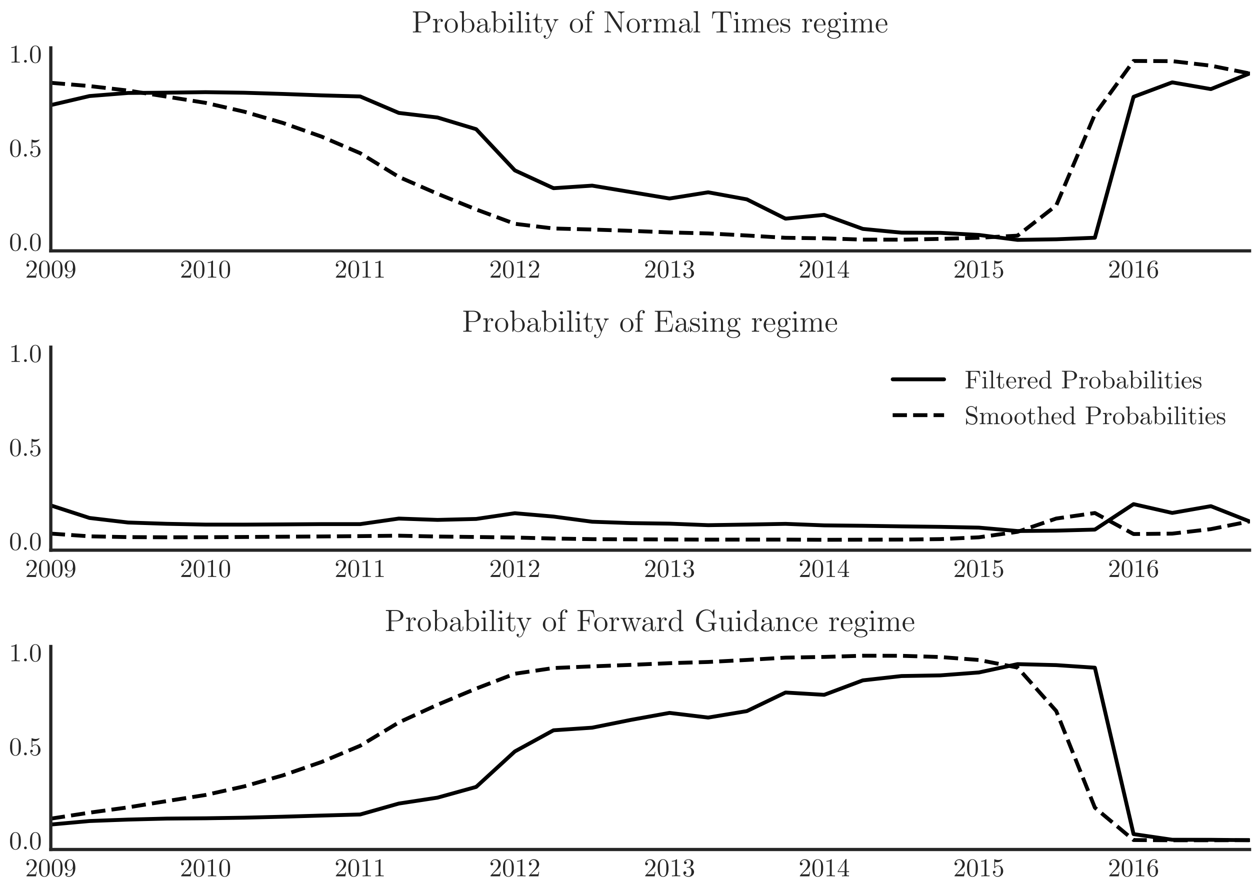
Notes: Figure displays the time series of the mean filtered estimates estimates of e_{Zt}, smoothed using a (centered) MA(8), for both the incomplete information (solid lines) and full information (dashed lines) specifications of the model.

Figure 3: Filtered Regime Probabilities



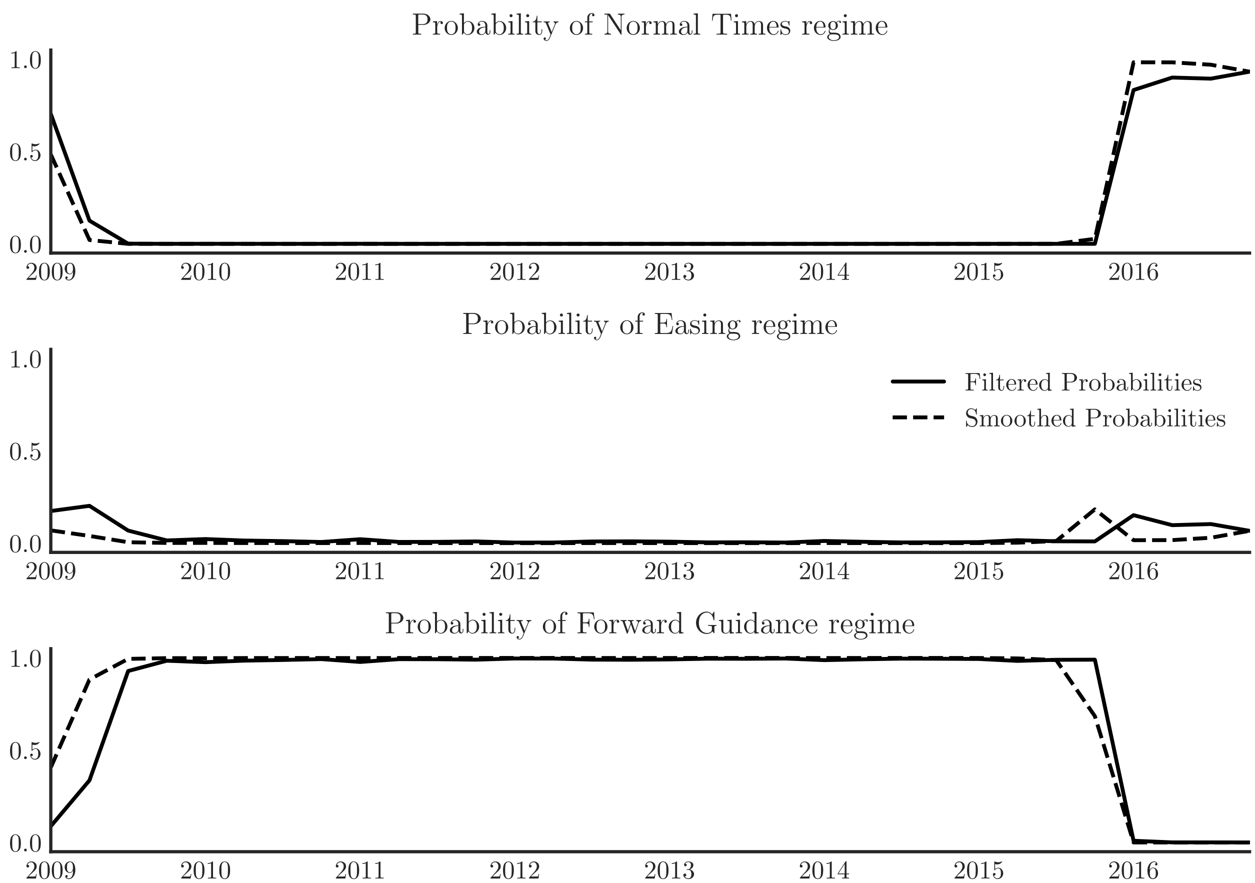
Notes: Figure displays the mean filtered probabilities of being in either the Normal Times (top panel), Easing (middle panel), or Forward Guidance (bottom panel) regime in the incomplete information (solid lines) or the full information (dashed lines) specification.

Figure 4: Filtered and Smoothed Regime Probabilities: Incomplete Information



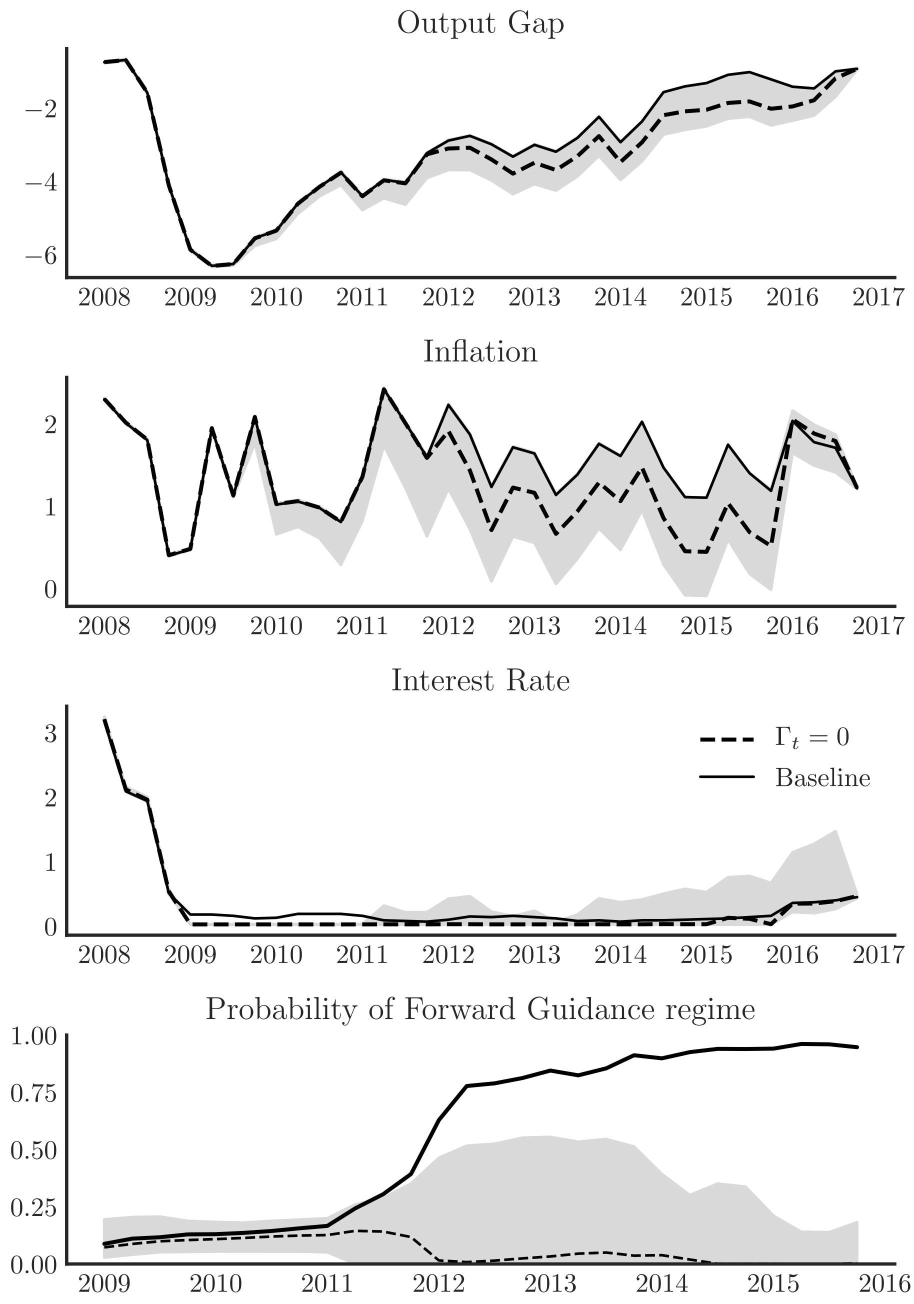
Notes: Figure displays the mean filtered (solid lines) and mean smoothed (dashed lines) probabilities of being in either the Normal Times (top panel), Easing (middle panel), or Forward Guidance (bottom panel) regime in the incomplete information specification of the model.

Figure 5: Filtered and Smoothed Regime Probabilties: No ZLB Constraint



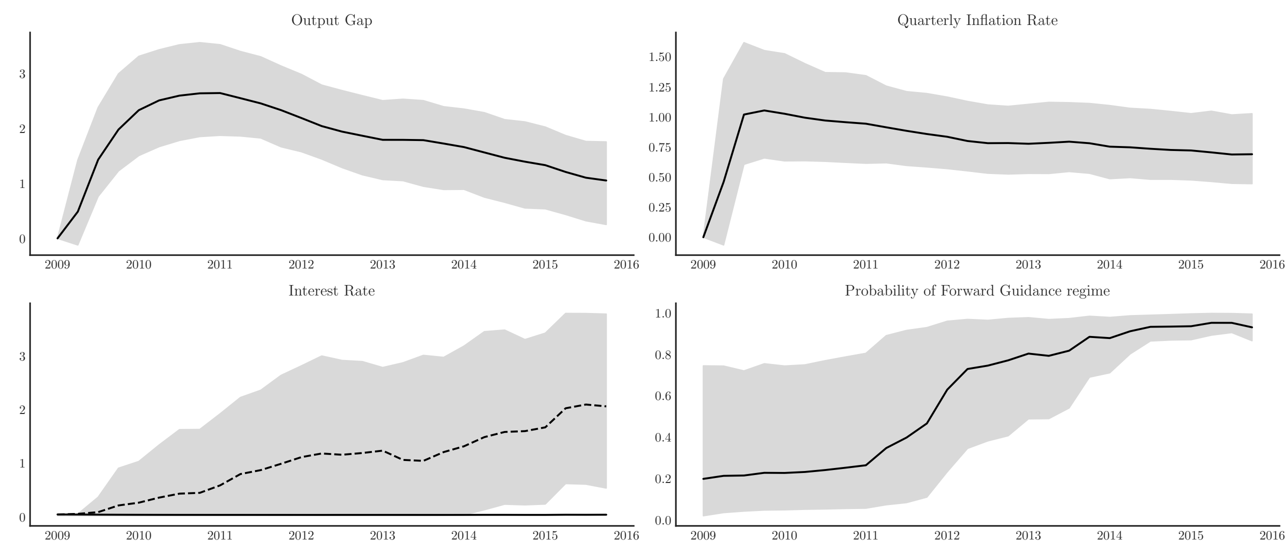
Notes: Figure displays the mean filtered (solid lines) and mean smoothed (dashed lines) probabilties of being in either the Normal Times (top panel), Easing (middle panel), or Forward Guidance (bottom panel) regime under incomplete information and ignoring the ZLB. The simulation uses the estimated parameters and states derived from the constrained version of the model.

Figure 6: The Effect of Forward Guidance



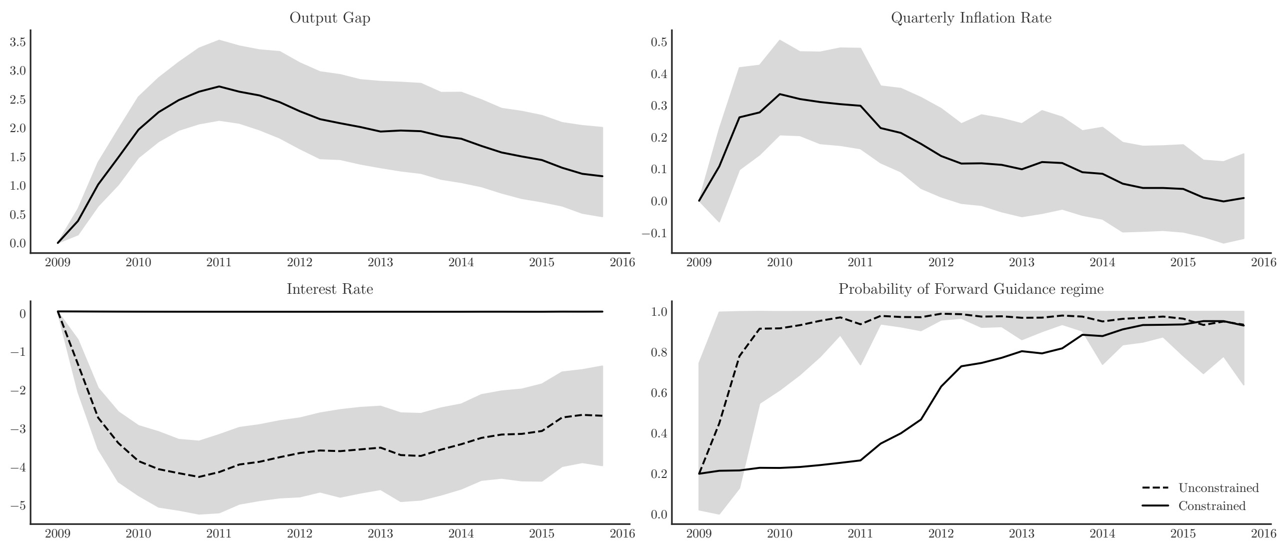
Notes: Figure shows trajectories of the output gap, inflation, interest rate, and probability of the Forward Guidance regime under the baseline estimation (solid line) and a counterfactual in which the policy rule remains in the Normal Times regime from 2009 onwards. The dashed line plots the median estimate, while the gray shaded regions denote the 90% credible bands.

Figure 7: The Effect of Full Information



Notes: The top two panels of the figure shows the difference in outcomes, in terms of percentage points, of the output gap and quarterly inflation rate, under full information and learning. The lower left panel shows the evolution of the federal funds under the learning model (solid line) and full information (dashed line) specifications, while the lower right panel shows the probabilities of the Forward Guidance regime in both of these specifications. The shaded regions indicate the 90 percent credible bands.

Figure 8: The Effect of the ZLB



Notes: The top two panels of the figure describe the difference in outcomes, in terms of percentage points, of the output gap and quarterly inflation in the unconstrained version of the model from the constrained version of the model. In both cases, the incomplete information specification is used. The lower left panel shows the evolution of the federal funds under the constrained version (solid line) and unconstrained version (dashed line), while the lower right panel shows the probabilities of the Forward Guidance regime in both of these model versions. The shaded regions indicate the 90 percent credible bands.

Online Appendix for
Monetary Policy Regimes, Credibility, and the Zero Lower Bound

Christopher Gust, Edward Herbst, and David Lopez-Salido

A Data

The data set spans from 1992:Q1 to 2016:Q4 (100 observations.)

1. **Output Gap**. Take Real Gross Domestic Product, (`FRED mnemonic: GDPC1`), call it GDP_t. Take the CBO's estimate of potential output (`FRED mnemonic: GDPPOT`), call it POT_t. Then output growth is defined as:
$$\text{Output Gap}_t = \log\left(\frac{GDP_t}{POT_t}\right)$$

2. **Inflation**. Take the Price Index for Personal Consumption Expenditures Less Food and Energy, (`FRED mnemonic: JCXFE`), call it P_t. Then annualized inflation is defined as:
$$\text{Inflation}_t = 4\log\left(\frac{P_t}{P_{t-1}}\right)$$

3. **Federal Funds Rate.**. Take the monthly federal funds rate, (`FRED mnenomic: FEDFUNDS`), averaged over the quarter, call it FFR_t. Then the federal funds are is defined as:
$$\text{Federal Funds Rate}_t = FFR_t.$$

B Equilibrium Conditions

The model's equilibrium conditions for C_t, λ_t, π_t, mc_t, Y_t, and R_t are:

$$\lambda_t = \frac{1}{C_t - \gamma C_{t-1}} \quad \text{(A-1)}$$

$$\lambda_t = \beta R_t \mathbb{E}\left[\lambda_{t+1}\pi_{t+1}^{-1}|\Omega_t\right] + \frac{\eta_t P_t}{B_{t+1}} \quad \text{(A-2)}$$

$$f_{\pi_t} = \beta\mathbb{E}\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{Y_{t+1}}{Y_t}f_{\pi_{t+1}}|\Omega_t\right] + \frac{\epsilon_t}{\varphi}\left[mc_t - \frac{\epsilon-1}{\epsilon}(1+\tau)\right] \quad \text{(A-3)}$$

$$f_{\pi_t} = \left[\frac{\pi_t}{\pi_{t-1}^{1-a}\bar{\pi}^a} - 1\right]\frac{\pi_t}{\pi_{t-1}^{1-a}\bar{\pi}^a} \quad \text{(A-4)}$$

$$\frac{1}{mc_t} = (1-\gamma)\lambda_t Z_t\left(\frac{Y_t}{Z_t}\right)^{\frac{\alpha}{1-\alpha}} \quad \text{(A-5)}$$

$$Y_t = C_t + \frac{\varphi}{2}\left(\frac{\pi_t}{\pi_{t-1}^{1-a}\bar{\pi}^a} - 1\right)^2 Y_t \quad \text{(A-6)}$$

$$\log R_t = \max\left[0, f_R\left(\mathbb{X}_t, \Gamma_{j_t}\right) + e_{Rt}\right] \quad \text{(A-7)}$$

$$f_R\left(\mathbb{X}_t, \Gamma_{j_t}\right) = \rho_R \log R_{t-1} + (1-\rho_R)\left[\gamma_\pi \log\left(\frac{\pi_t}{\bar{\pi}}\right) + \gamma_y \log\left(\tilde{y}_t\right)\right] + \Gamma_{j_t} \quad \text{(A-8)}$$

where $\tilde{y}_t = \frac{Y_t}{Z_t}$ and we normalize $b_{t+1} = \frac{B_{t+1}}{Z_t P_t} = \frac{1}{1-\gamma}$ and $\chi_0 = \frac{1-\alpha}{1-\gamma}$. We also set $(1+\tau) = \frac{\epsilon-1}{\epsilon}$ so that the steady state value of mc_t equals one. In the non-stochastic steady state, $\pi = \bar{\pi}$ so that $\tilde{y} = \tilde{c}$ where $\tilde{c}_t = \frac{C_t}{Z_t}$. Defining $\tilde{\lambda}_t = Z_t \lambda_t$, it then follows from the fact that real marginal cost is equal to one in steady state as well as equations (A-1) and (A-1) that the steady state value of $\tilde{y}_t$ is equal to one.