

# Short-term Planning, Monetary Policy, and Macroeconomic Persistence

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## Abstract

This paper uses aggregate data to estimate and evaluate the finite horizon (FH) model recently proposed by [Woodford \(2018\)](#). The model outperforms rational expectations versions of the New Keynesian (NK) model commonly used in empirical applications as well as other behavioral NK models. The better fit of the FH model reflects that it can induce slow-moving trends in key endogenous variables which deliver substantial persistence in output and inflation dynamics. The FH model does so without resorting to additional features such as habit persistence and price contracts indexed to lagged inflation. Our empirical estimates indicate that the planning horizons of most households and firms are less than two years which considerably dampens the effects of expected future changes of monetary policy on the macroeconomy.

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# 1 Introduction

Macroeconomists have long understood the important role that expectations play in determining the effects of monetary policy. Although it is common to analyze the effects of monetary policy assuming expectations are formed rationally, a growing literature inspired by experimental evidence on human judgement and the limits of cognitive abilities has emphasized the policy implications of macroeconomic models in which expectations are formed in a way that is consistent with this evidence.<sup>1</sup> This literature has emphasized several advantages of the approach including the more realistic dynamics these models can generate in response to changes in monetary policy that affect future policy rates. More specifically, advocates of behavioral macro models point to a “forward guidance puzzle” in which a credible promise to keep the policy rate unchanged in the distant future has unreasonably large effects on current inflation and output in New Keynesian (NK) models with rational agents. In contrast, they show that NK models in which expectations are consistent with behavioral evidence do not display such a puzzle.<sup>2</sup> Although results such as these suggest that behavioral macro models are a promising alternative to those with rational expectations, it remains an open question whether these models can be developed into empirically-realistic ones capable of providing quantitative guidance for monetary policy.

In this paper, we take a step towards addressing this question by estimating several New Keynesian (NK) models with behavioral features and assessing their ability to account for fluctuations in inflation, output, and interest rates in the United States. Our analysis suggests that the finite-horizon (FH) approach developed in a recent contribution by [Woodford \(2018\)](#) is a promising framework for explaining aggregate data and analyzing monetary policy. A chief advantage of the FH approach that we identify is its ability to deliver persistent movements in aggregate data, as the behavioral assumptions that underlie it give rise to a trend-cycle decomposition in which endogenous persistence arises from slow-moving trends.

As argued in [Schorfheide \(2013\)](#), one of the key challenges in developing empirically-realistic macroeconomic models is that there is substantial low frequency variation in macroeconomic data that makes accurate inferences about business cycle fluctuations difficult. A number of researchers have attempted to address this concern by incorporating exogenous shock processes to capture movements in trends; however, this approach can lead to movements in trends that are largely exogenous and unrelated to those driving business cycle fluctuations.<sup>3</sup> In contrast, in the finite-horizon approach of [Woodford \(2018\)](#), cyclical fluctuations are an important determinant of the

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<sup>1</sup>Recent contributions include [Gabaix \(2018\)](#), [Garcia-Schmidt and Woodford \(2019\)](#) and [Farhi and Werning \(2018\)](#), and [Angeletos and Lian \(2018\)](#). This literature is closely related to earlier work in models with boundedly rational agents; see, for example, [Sargent \(1993\)](#).

<sup>2</sup>See [Del Negro, Giannoni, and Patterson \(2012\)](#) and [McKay, Nakamura, and Steinsson \(2016\)](#) for a discussion of the forward guidance puzzle. While the behavioral NK literature has emphasized the importance of incorporating boundedly rational agents into monetary models, others have emphasized the assumption that households and firms may not view promises about future rates as perfectly credible. In an estimated model, [Gust, Herbst, and Lopez-Salido \(2018\)](#), for example, show that imperfect credibility was an important reason why the Federal Reserve’s forward guidance was less effective than otherwise.

<sup>3</sup>See [Canova \(2014\)](#) for a discussion of the issue and approaches in which the trends are modeled exogenously and independently from the structural model used to explain business cycle fluctuations.

model's trends, and one of the key contributions of this paper is to empirically evaluate this feature of the model.

To understand how the FH model in [Woodford \(2018\)](#) gives rise to a theory by which the cycle contributes to slow-moving trends, it is useful to review key features of the approach. The backbone of the model is still New Keynesian, as monopolistically-competitive firms set prices in a staggered fashion and households make intertemporal consumption and savings decisions. However, a household in making those decisions and a firm in setting its price over multiple periods do so based on plans made over only a finite horizon despite their decisions having implications that last beyond that short horizon. In doing so, households and firms are still quite sophisticated in that their current decisions involve making forecasts and fully-state contingent plans over their finite but limited horizons.

Households and firms, as in a standard NK model, are still infinitely-lived and need to look into the far distant future to make their current decisions. A key assumption of the FH approach is that households and firms are boundedly rational in thinking about the value functions which determine the continuation values of their plans over their infinite lifetimes. Instead of viewing their value functions as fully state-contingent as it would be if their expectations were rational, agents' beliefs about their value functions are coarser in their state dependence. Moreover, households and firms do not use the relationships in the model to infer their value functions; instead, they update the continuation values associated with their finite-horizon plans based on past data that they observe.<sup>4</sup>

Because of this decision-making process, the FH model gives rise to households and firms who are forward-looking in thinking about events over their planning horizon but are backward looking in thinking about events beyond that point. If the planning horizons of households and firms becomes very long, the dynamics of the FH model mimics those of a standard NK model so that the backward-looking behavior becomes irrelevant. However, to the extent that a significant fraction of agents have short planning horizons, the dynamics of the model are notably different from those of a more standard NK model. In particular, as emphasized in [Woodford \(2018\)](#), changes in future policy rates are not as effective in influencing current output and inflation. Future changes in the output gap also have a much smaller effect on current inflation.

Most importantly, when a material fraction of agents have short-planning horizons, the model is capable of generating persistence endogenously through the way agents update their beliefs about their value functions. Because of this feature, the model's equilibrium dynamics can be decomposed into a cyclical component governed by the agents' forward looking behavior and a trend component governed by the way agents update their beliefs about their value functions. Because agents update their beliefs in a backward-looking manner, the model is capable of generating substantial persistence in output and inflation arising from slow-moving trends. For instance, in line with empirical evidence, we show that the model is capable of generating substantial inflation persistence and a hump-shaped response of output following a monetary policy shock. Notably, it

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<sup>4</sup>[Woodford \(2018\)](#) motivates such decision making based on the complex intertemporal choices made by sophisticated artificial intelligence programs.

does so in the absence of incorporating habit persistence in consumption, price-indexation contracts tied to lagged inflation, or adjustment costs to investment.<sup>5</sup>

We employ Bayesian methods to estimate the FH model as well as other behavioral macro models using U.S. quarterly data on output, inflation, and interest rates from 1966 until 2007. Besides comparing the FH model’s performance to other behavioral macro models, we also compare its performance to a hybrid NK model that incorporates habit persistence and price-indexation contracts tied to lagged inflation. Because there is notable low frequency variation in the variables over the sample period that we estimate, we also compare the FH model’s ability to fit the data relative to a NK model in which there are exogenous and separate processes used to model the trends in output, inflation, and nominal short-term interest rates.

Regarding the estimation of the FH model, we find that we can reject parameterizations in which there is a considerable fraction of agents with long planning horizons including the standard NK model in which agents are purely forward looking. Our mean estimates suggest that about 50 percent of households and firms have planning horizons that include only the current quarter, 25 percent have planning horizons of two quarters, and only a small fraction have a planning horizon beyond 2 years. Thus, our estimates imply that there is a substantial degree of short term planning. Our evidence is also consistent with agents updating their value functions slowly in response to recently observed data so that the model’s implied trends also adjust slowly. We show that because of this feature the model can account for the substantial changes that occurred to trend inflation and trend interest rates in the 1970s and 1980s. Interestingly, the model’s measure of trend inflation, for instance, displays similar movements to a measure of longer-term inflation expectations coming from the Survey of Professional Forecasters.

We also show that the FH model fits the observed dynamics of output, inflation, and interest rates better than the hybrid NK model. This better fit reflects both the endogenous persistence generated by agents’ learning about their value functions as well as the reduced degree of forward-looking behavior associated with short-term planning horizons. Because of the model’s ability to generate slow moving trends, its goodness of fit measure is substantially better than the behavioral macro models of [Angeletos and Lian \(2018\)](#) and [Gabaix \(2018\)](#). The model also fits moderately better than a NK model that incorporates exogenous and separate trends in output, inflation, and interest rates. Overall, we view these results as suggesting that the FH approach is a parsimonious and fruitful way to understand business cycle fluctuations in the context of slow moving trends.

The rest of the paper is structured as follows. The next section describes the FH model of [Woodford \(2018\)](#) paying particular attention to the role of monetary policy and the model’s trend-cycle decomposition. Section 3 analyzes the dynamic properties of the model further and shows that the model is capable of generating realistic dynamics following a monetary policy shock. Section 4 presents the estimation results of the FH model, while Section 5 compares the fit of the FH model to the other models that we estimate. Section 6 concludes and offers directions for further research.

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<sup>5</sup>These mechanisms are often described as forms of generating “intrinsic persistence” in output and inflation (e.g., [Smets and Wouters \(2007\)](#) and [Christiano, Eichenbaum, and Evans \(2005\)](#)). [Sims \(1998\)](#) is an important earlier contribution to the discussion of issues related to modeling persistence in the context of macroeconomic models.

## 2 A Finite-Horizon-Planning NK Model

We now present a description of the key structural relationships of the finite-horizon model that we estimate. The derivation of these expressions can be found in [Woodford \(2018\)](#).

To help motivate the finite-horizon approach, it is helpful to first review the structural relationships from the canonical NK model.<sup>6</sup> In that model, aggregate output  $y_t$  and inflation  $\pi_t$  (expressed in log-deviations from steady state) evolve according to the following expressions:

$$y_t - \xi_t = E_t[y_{t+1} - \xi_{t+1}] - \sigma [i_t - E_t(\pi_{t+1})] \quad (1)$$

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa(y_t - y_t^*) \quad (2)$$

where  $E_t$  denotes the model-consistent expectations operator conditional on available information at time  $t$ ,  $\xi_t$  is a demand or preference shock and  $y_t^*$  is exogenous and captures the effects of supply shocks. The parameters  $\beta$ ,  $\sigma$ , and  $\kappa$  are the discount factor, the inverse of the household's relative risk aversion, and the slope of the inflation equation with respect to aggregate output. The parameter  $\kappa$  itself is a function of structural model parameters including the Calvo parameter governing the frequency of price adjustment and the elasticity of output to labor in a firm's production function. To close the model, a central bank is assumed to follow an interest-rate ( $i_t$ ) policy rule:

$$i_t = \phi_\pi \pi_t + \phi_y y_t + i_t^*, \quad (3)$$

where  $\phi_\pi > 0$ ,  $\phi_y > 0$ , and  $i_t^*$  as an exogenous monetary policy surprise. These three equations can be used to characterize the equilibrium for output, inflation and the short-term interest rate in the canonical NK model.

The finite-horizon model in [Woodford \(2018\)](#) maintains two key ingredients of the canonical model. In particular, monopolistically-competitive firms set prices in a staggered fashion according to [Calvo \(1983\)](#) contracts and households make intertemporal choices regarding consumption and savings. However, the finite-horizon approach departs from the assumption that households and firms formulate complete state-contingent plans over an infinite-horizon. Instead, infinitely-lived households and firms make state-contingent plans over a fixed  $k$ -period horizon taking their infinite-horizon continuation values as given. While households and firms are sophisticated about their plans over this fixed horizon, they are less sophisticated in thinking about continuation values. In particular, [Woodford \(2018\)](#) assumes that agents are not able to use their model environment to correctly deduce their value functions and how they differ across each possible state. Instead, the value function is coarser in its state dependence. Agents update their beliefs over time about their value functions as they gain information about them as the economy evolves.

This introduces a form of bounded rationality in which agents choose a plan at date  $t$  over the next  $k$  periods but only implement the date  $t$  part of the plan. To make their decisions about date  $t$  variables, households and firms take into account the state contingencies that could arise over

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<sup>6</sup>See [Woodford \(2003\)](#) or [Galí \(2008\)](#) for the derivations of the canonical NK model.

the next  $k$  periods, working backwards from their current beliefs about their value-functions.<sup>7</sup> In period  $t + 1$ , an agent will not continue with the plan originally chosen at time  $t$  but will choose a new plan and base their time  $t + 1$  decisions on that revised plan. An agent will also not necessarily use the same value-function that she used at date  $t$ , as an agent may update her value function for decisions at date  $t + 1$ .

The model allows for heterogeneity over the horizons with which firms and households make their plans. In the presence of this heterogeneity, [Woodford \(2018\)](#) is able to derive a log-linear approximation to the finite-horizon model whose aggregate variables evolve in a manner resembling the equilibrium conditions of the canonical NK model. In particular, aggregate output and inflation satisfy:

$$y_t - \xi_t - \bar{y}_t = \rho E_t[y_{t+1} - \xi_{t+1} - \bar{y}_{t+1}] - \sigma [i_t - \bar{i}_t - \rho E_t(\pi_{t+1} - \bar{\pi}_{t+1})] \quad (4)$$

$$\pi_t - \bar{\pi}_t = \beta \rho E_t[\pi_{t+1} - \bar{\pi}_{t+1}] + \kappa(y_t - y_t^* - \bar{y}_t) \quad (5)$$

$$i_t - \bar{i}_t = \phi_\pi(\pi_t - \bar{\pi}_t) + \phi_y(y_t - \bar{y}_t) + i_t^* \quad (6)$$

Three elements stand out about aggregate dynamics of the finite-horizon model. First, there is an additional parameter,  $0 < \rho < 1$ , in front of the expected future values for output and inflation. Second, aggregate output and inflation are written in deviations from endogenously-determined “trends”; the trends vary over time and are represented by a “bar” over a variable. Finally, monetary policy responds to the deviation of inflation and output from their trends and allows for a time-varying intercept ( $\bar{i}_t$ ). We discuss each of these elements in more detail below.

## 2.1 Microeconomic Heterogeneity and Short-term Planning

The parameter  $\rho$  is an aggregate parameter reflecting that planning horizons differ across households and firms. To understand this, let  $\omega_j$  and  $\tilde{\omega}_j$  be the fraction of households and firms, respectively, that have planning horizon  $j$  for  $j = 0, 1, 2, \dots$ ; the sequences of  $\omega$ 's satisfy  $\sum_j \omega_j = \sum_j \tilde{\omega}_j = 1$ . The parameter  $\rho$  satisfies  $\omega_j = \tilde{\omega}_j = (1 - \rho)\rho^j$  where  $0 < \rho < 1$ . In this case, aggregate spending and inflation are themselves the sum of spending and pricing decisions over the heterogeneous households and firms. As a result,  $y_t = \sum_j (1 - \rho)\rho^j y_t^j$  and  $\pi_t = \sum_j (1 - \rho)\rho^j \pi_t^j$ , where  $y_t^j$  denotes the amount of spending of a household with planning horizon  $j$  and  $\pi_t^j$  denotes the inflation rate set by a firm with planning horizon  $j$ .<sup>8</sup>

The parameter  $\rho$  governs the distribution of planning horizons agents have in the economy and has important implications for aggregate dynamics. A relatively low value of  $\rho$  implies that the fraction of agents with a short planning horizon is relatively high. And, as a consequence, the dynamics characterizing aggregate output and inflation are less “forward-looking” than in the canonical model. In fact, as  $\rho$  approaches zero, the expressions governing aggregate demand and

<sup>7</sup>[Woodford \(2018\)](#) motivates this approach based on sophisticated, artificial intelligence programs constructed to play games like chess and go.

<sup>8</sup>As discussed in [Woodford \(2018\)](#), with an infinite number of types the existence of the equilibrium requires that these (infinite) sums determining aggregate output and inflation converge.

inflation become increasingly similar to a system of static equations. When  $\rho \rightarrow 1$ , there are an increasing number of households and firms with long planning horizons and the aggregate dynamics become like those of the canonical model in which agents have rational expectations. Because of its prominent role in affecting the cyclical component of aggregate dynamics, one aim of this paper is to estimate the value of  $\rho$  and see how much short-term planning by households and firms is necessary to explain the observed persistence in output, inflation, and interest rates.

Woodford (2018) also emphasizes the important role that  $\rho < 1$  plays in overcoming the forward guidance puzzle inherent in the canonical NK model – i.e., the powerful effects on current output and inflation of promises about future interest rates. To see this, suppose, for simplicity, that the trends components of the model are exogenous (an aspect that we will be relaxed later on in the paper). Accordingly, the system of equations (4) - (6) can be used to characterize the evolution of the “cyclical” components of output, inflation and the short-term interest rate. Solving forward the equation (4), the cyclical component of aggregate output can be written as:

$$\tilde{y}_t = -\sigma \sum_{s=0}^{\infty} \rho^s E_t(\tilde{i}_{t+s} - \tilde{\pi}_{t+s+1}) - \sigma(1 - \rho) \sum_{s=0}^{\infty} \rho^s E_t \tilde{\pi}_{t+s+1} \quad (7)$$

where the symbol “ $\sim$ ” represents the cyclical component of the variables (i.e., the value of the variable in deviation from its trend:  $\tilde{x}_t = x_t - \bar{x}_t$ ). The expression above differs in two important ways from the aggregate output equation in the canonical NK . First, current (cyclical) output depends on the “discounted future” path of the (cyclical) short-term real rates, and the geometric weights of future rates on current output are a function of the parameter  $\rho$ . This stands in sharp contrast with the canonical model in which there is no discounting in the sum of future short-term (cyclical) interest rates (i.e., the case of  $\rho \rightarrow 1$ ). This implies that the cyclical output does not respond equally to a change in the interest rates occurring next period (i.e.,  $t + 1$ ) than to a change happening further in the future (i.e., in period  $t + k$ ). From the expression above, it follows that the effect on current (cyclical) output of a change in the cyclical short-term rate in period  $t + k$  is given by the amount  $-\sigma\rho^k$ ; a value that now approaches to zero as the change in the short-term rate is far into the future or if the value of the parameter  $\rho$  is relatively close to zero – i.e., a substantial fraction of households have relatively short-horizon planning.<sup>9</sup>

The second term in the right-hand side of expression (7) is also novel and reflects a form of misspricing of future rates associated with agents planning only over short-horizons. This term resembles the form of “money illusion” considered by Modigliani and Cohn (1979) and recently invoked by Brunnermeier and Julliard (2008). That is, according to these authors agents fail to

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<sup>9</sup>Likewise, the aggregate supply (expression (5)) is affected by the parameter  $\rho$ . Forward iteration using expression (5) yields:

$$\tilde{\pi}_t = \kappa \sum_{s=0}^{\infty} (\beta\rho)^s E_t \tilde{y}_{t+s}$$

where  $\beta\rho < \beta$ . This implies that current inflation depends on current and expected future values of the output gap, but relative to the baseline model the effects of far-future gaps on inflation diminish substantially with the parameter  $\rho$ .



distinguish between real and nominal rate of returns. Thus, they mistakenly attribute a decrease (increase) in inflation to a decline (increase) in real returns. In the current model, at the aggregate level, the short-term planning creates a similar misspricing as reflected by the term  $\psi_t = -\sigma(1 - \rho) \sum_{s=0}^{\infty} \rho^s E_t \tilde{\pi}_{t+s+1}$  in expression (7).<sup>10</sup> That is, the misspricing term ( $\psi_t$ ) and the cyclical demand ( $\tilde{y}_t$ ) are increasing if inflation (in deviation from its trend) is expected to decline in the future. This point is important, because this negative misspricing error applies to the cyclical components of the variables. However, if agents have a trend level of inflation jointly and endogenously determined (and hence affected by the central bank) this is not necessarily true. The next section brings the analysis of the trends into the description of the aggregate equilibrium.

## 2.2 A Theory-Based Trend-Cycle Decomposition

Expressions (4) and (5) describe the evolution of aggregate output and inflation in deviation from trend. The trend variables in equations (4)-(6) themselves are in deviation from nonstochastic steady state so that these trends can reflect very low frequency movements in these variables but not those associated with a change in the model's steady state. Instead, these time-varying trends reflect changes in agents' beliefs about the longer-run continuation values associated with their plans. Because agents update their continuation values based on observation of past data, this updating can induce persistence trends in output, inflation, and the short-term interest rate. We now turn to discussing how the finite-horizon approach in [Woodford \(2018\)](#) leads to such a trend-cycle decomposition.

To understand how do agents in the model parse trend from cycle, it is necessary to describe the nature of the value functions of households and firms. In the case of a household, its value function depends on its wealth or asset position and the derivative of the value function with respect to wealth is a key determinant of a household's first order conditions and thus their optimal decisions. This derivative itself is a function and determines the marginal (continuation) value to a household of holding a particular amount of wealth. Unlike under rational expectations, this function is not fully state-contingent and it is assumed that agents can not deduce it using the relationships of the model. Instead, they update the parameters governing the marginal value of wealth based on past experience. More specifically, the marginal value of wealth in log-linear form consists of an intercept term and a slope coefficient on household wealth. Under constant-gain learning, [Woodford \(2018\)](#) proves that only the intercept-term needs to be updated, as the slope coefficient can be shown to converge to a constant. Constant-gain learning implies that a household updates the intercept-term of her marginal value of wealth according to:

$$v_{t+1} = (1 - \gamma)v_t + \gamma v_t^{est}, \tag{8}$$

where  $v_t$  denotes the log-linearized intercept of the marginal value of wealth at date  $t$ . Also, the

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<sup>10</sup> Interestingly, a similar measure was considered by [Brunnermeier and Julliard \(2008\)](#) in their analysis of proxy misspricing of the price-to-rent ratio by non-rational agents.



constant-gain parameter,  $\gamma$ , satisfies  $0 < \gamma < 1$  and determines how much weight a household put on the current estimate of this intercept in the updating step. The variable  $v_t^{est}$  denotes an updated (estimate for the) intercept that a household computes based on information acquired at date  $t$ . Through recursive substitution of expression (8), one can see that a household's current estimate of its value function,  $v_t$ , is a weighted sum of all the past values of  $v_t^{est}$  with distant past values getting more weight the larger is  $(1 - \gamma)$ .

Taking  $v_t$  as given, a household can also compute a new estimate of the marginal (continuation) value associated with its optimal plans. Using its current marginal continuation value, a household chooses its optimal plans over its finite-horizon. Woodford (2018) shows that the intercept term of its new estimate satisfies:

$$v_t^{est} = y_t - \xi_t + \sigma\pi_t. \quad (9)$$

Thus, the updated intercept term depends on current spending and current inflation. Combining this expression with equation (8) yields an expression in which the marginal value of wealth depends on all past values of  $y_t$ ,  $\xi_t$ , and  $\pi_t$ .

We turn now to the discussion of how firms update their value function. A firm who has the opportunity to reset its price makes a similar finite-horizon plan in choosing its price and this decision depends on the continuation value associated with that plan. This value function reflects that a firm chooses its price to maximize the expected discounted stream of profits. In doing so, a firm takes into account that with probability " $\alpha$ " it will not have the opportunity to reset its price; this makes its price-setting decision a dynamic one. A firm's optimal plan for setting its price depends on the continuation value. More specifically, the optimal condition associated with this plan depends on the derivative of a firm's value-function with respect to its (relative) price. This determines the marginal continuation value associated with a firm's price-setting decision in terms of its effect on firm's stream of profits.

As was the case for households, this marginal continuation value is not fully state-contingent and it is assumed that firms can not deduce its nature using the relationships of the model. Instead, the parameters governing the marginal continuation value of their price are updated based on past experience. In its log-linear form, these parameter consist of an intercept term and a slope coefficient on a firm's relative price. Similar to households, only the intercept term,  $\tilde{v}_t$ , is updated by firms. This intercept term is updated according to:

$$\tilde{v}_{t+1} = (1 - \tilde{\gamma})\tilde{v}_t + \tilde{\gamma}\tilde{v}_t^{est}, \quad (10)$$

where  $\tilde{\gamma}$  is the constant-gain learning parameter and  $\tilde{v}_t^{est}$  is a new estimate of a firm's marginal continuation value. Woodford (2018) shows that  $\tilde{v}_t^{est}$  satisfies:

$$\tilde{v}_t^{est} = (1 - \alpha)^{-1}\pi_t. \quad (11)$$

Notice that (to a log-linear approximation) the best estimate of the intercept characterizing the evolution of each firm's value function depends on aggregate inflation. This results from the fact

that firms reoptimizing prices in any given period choose a (relative) price that differs from the economy average price in previous period and hence it can be related to the aggregate price change (i.e., inflation). The effect of inflation on the best estimate of the intercept of the firm's value function depends on the average price duration that in the model corresponds to the expression:  $(1 - \alpha)^{-1}$ .

Because the the longer-run continuation values of households and firms reflect averages of past values of spending and inflation, they can induce slow moving trends in the aggregate economy. More concretely, the finite-horizon model implies that output and inflation can be decomposed into a cyclical component (denoted using a tilde) and trend component (denoted using a bar) so that  $y_t = \tilde{y}_t + \bar{y}_t$  and  $\pi_t = \tilde{\pi}_t + \bar{\pi}_t$ . The trend components represent how the spending and pricing decisions are affected by  $v_t$  and  $\tilde{v}_t$ , while the cyclical component represents these decisions in the absence of any changes in  $v_t$  and  $\tilde{v}_t$ . Because a household is (still) forward-looking, its plan for spending in future periods as well as the plan's continuation value matter for its current spending decision. Similarly, a firm with the opportunity to reset its price is forward looking so that  $\tilde{v}_t$  matters for that decision. Averaging across the different household types, [Woodford \(2018\)](#) shows that the effect of  $v_t$  on aggregate spending is given by:

$$\bar{y}_t = \frac{-\sigma}{1 - \rho} (\bar{i}_t - \rho \bar{\pi}_t) + v_t, \quad (12)$$

where  $\bar{i}_t$  is the trend interest rate discussed further below. Similarly, averaging across firms with different planning horizons, the effect of  $\tilde{v}_t$  on average price inflation is given by:

$$\bar{\pi}_t = \frac{\kappa}{1 - \beta\rho} \bar{y}_t + \frac{(1 - \rho)(1 - \alpha)\beta}{1 - \beta\rho} \tilde{v}_t. \quad (13)$$

Holding fixed the trend interest rate, equations (12) and (13) relate trend output and inflation to the longer-run continuation values of households and firms. These continuation values, as reflected in  $v_t$  and  $\tilde{v}_t$ , in turn depend on the entire past history of aggregate spending and inflation and thus the model is capable of generating substantial persistence in output and inflation trends. Importantly, as indicated by the presence of  $\bar{i}_t$  in equation (12), trend output and inflation depend on agents' views about monetary policy in the long run, which we now specify.

### 2.3 Monetary Policy

We depart from [Woodford \(2018\)](#) by allowing monetary policy to respond to movements in trends differently than cyclical fluctuations. In particular, the intercept term in equation (6) is specified as:

$$\bar{i}_t = \bar{\phi}_\pi \bar{\pi}_t + \bar{\phi}_y \bar{y}_t. \quad (14)$$

With  $\phi_\pi = \bar{\phi}_\pi$  and  $\phi_y = \bar{\phi}_y$ , the response of monetary policy to the trend and the cycle is the same. In that case, the rule is exactly the same as the one in [Woodford \(2018\)](#), and we can write the rule as in equation (3), which expresses the rule in terms of deviations of aggregate output and inflation

from their steady state values. However, monetary policy may respond differently to persistent deviations of inflation or output from their steady state values than more transitory fluctuations and to capture this possibility we allow the trend response coefficients,  $\bar{\phi}_\pi$  and  $\bar{\phi}_y$ , to differ from their cyclical counterparts. In our empirical analysis, we evaluate whether such a response is a better characterization of monetary policy than the case in which monetary policy responds to the trend and cycle equiproportionately.

### 3 Short-Term Planning and Macroeconomic Persistence

In this section, we investigate the model's trend-cycle decomposition more thoroughly and show how it induces persistent movements in output and inflation following a monetary policy shock. We begin by showing that in the finite-horizon model, cyclical fluctuations are independent from the trend. However, the trends depend on the cycle and thus on monetary policy.

#### 3.1 Trend-Cycle Decomposition and Monetary Policy

To see that the cycle is independent of the trend, note that equations (4)-(6) are block recursive when we express output, inflation, and the policy rate as deviations from trends. Specifically, after substituting out the policy rate deviation using the interest-rate rule, the remaining two equations yield:

$$\tilde{x}_t = \rho M \cdot E_t[\tilde{x}_{t+1}] + N \cdot u_t, \quad (15)$$

where  $\tilde{x}_t = (\tilde{y}_t - \xi_t, \tilde{\pi}_t)'$  and  $u_t = (i_t^* + \phi_y \xi_t, \xi_t - y_t^*)'$ . Also,  $M$  and  $N$  are 2-by-2 matrices whose elements depend on the model's structural parameters including the rule parameters,  $\phi_\pi$  and  $\phi_y$ . (The appendix shows the elements of  $M$  and  $N$  as a function of the model's parameters.) This system can be used to solve for the cyclical variables,  $\tilde{x}_t$ , as a function of the economy's shocks,  $u_t$ , independently of the trends for output, inflation, or the policy rate. As a result, the cyclical variables do not depend on the long-run response of monetary policy to the trends (i.e.,  $\bar{\phi}_\pi$  and  $\bar{\phi}_y$ ).

The trends, however, depend on the cycle. To see that, expressions (12) and (13) can be used to solve for  $\bar{y}_t$  and  $\bar{\pi}_t$  as a function of  $v_t$  and  $\tilde{v}_t$ :

$$\bar{x}_t = (1 - \rho)\Theta V_t, \quad (16)$$

where we have substituted out  $\tilde{v}_t$  using equation (14),  $\bar{x}_t = (\bar{y}_t, \bar{\pi}_t)'$ , and  $V_t = (v_t, \tilde{v}_t)'$ . The 2-by-2 matrix,  $\Theta$ , is shown in the appendix and depends on structural model parameters that include  $\bar{\phi}_\pi$  and  $\bar{\phi}_y$ . Thus, the trends for output and inflation depend importantly on how monetary policy responds to their movements, since monetary policy affects agents' longer-run continuation values.

To express the trends,  $\bar{x}_t$ , as a function of the cycle, it is convenient to rewrite the laws of

motion for the intercepts of the marginal value-function as:

$$V_t = (I - \Gamma)V_{t-1} + \Gamma\Phi x_{t-1}, \quad (17)$$

where  $x_{t-1} = (y_t - \xi_t, \pi_t)'$ , and  $\Gamma$  and  $\Phi$  are 2-by-2 matrices shown in the appendix. Importantly, they do not depend on the monetary policy rule parameters. Combining expression (17) with equation (16) yields:

$$\bar{x}_t = \Lambda\bar{x}_{t-1} + (1 - \rho)\gamma Qx_{t-1}, \quad (18)$$

where  $\Lambda = \Theta(I - \Gamma)\Theta^{-1}$  and  $Q = \Theta\Gamma\Phi$  are also 2-by-2 matrices shown in the appendix. Using  $\tilde{x}_t = x_t - \bar{x}_t$ , we can rewrite this expression so that the trends for output and inflation are a function of the past cyclical values for these variables:

$$\bar{x}_t = [\Lambda + (1 - \rho)\gamma Q]\bar{x}_{t-1} + (1 - \rho)\gamma Q\tilde{x}_{t-1} \quad (19)$$

The aggregate equilibrium consists of the forward-looking system given by expression (15) characterizing the cycle and a backward-looking system given by expression (19) characterizing the trends. Because the cycle is independent of agents' beliefs about the trends, one can determine the cycle by solving the system in expression (15) for  $\tilde{y}_t$  and  $\tilde{\pi}_t$  and then using these values to determine the trends using expression (19).

*Discussion.* So far, our analysis of the model's trend-cycle decomposition has followed [Woodford \(2018\)](#). Here we extend the analysis of [Woodford \(2018\)](#). First, while [Woodford \(2018\)](#) shows that the stability of the trends depends on  $0 < \gamma < 1$  and  $0 < \tilde{\gamma} < 1$ , we show that a modified Taylor principle is necessary for stability of the forward-looking system. Second, in the appendix, we provide analytical expressions for the matrices,  $\Lambda$  and  $Q$ , allowing for a better understanding of the model's trend-cycle decomposition.

For the stability of the system given by expression (15), the Taylor principle needs to be modified. As shown in the appendix, the modified Taylor principle for the FH model is:

$$\left(\frac{1 - \rho\beta}{\kappa}\right)\phi_y + \phi_\pi > \rho. \quad (20)$$

Accordingly, the canonical model is a special case in which  $\rho = 1$  and in general the Taylor principle is relaxed relative to the canonical model when agents have finite horizons. Moreover, the Taylor principle depends on how policy responds in the short run and not on how policy responds to fluctuations in trends.

The impact the cycle has on trend inflation depends on the planning horizon of agents, the speed at which they update their value functions, and how responsive policy is to movements in trend variables. As  $\rho$  increases toward one, agents have long planning horizons and as indicated by equation (19), the trends no longer depend on the cycle. In fact, the trends become constants at their steady state values and the model's cyclical dynamics mimic those of the canonical NK model.

With  $\gamma = \tilde{\gamma}$ , households and firms update their value-functions at the same rate. As shown in the appendix, this is an interesting case because the feedback matrix  $\Lambda$  becomes the scalar,  $1 - \gamma$ , and the matrix  $Q$  is independent of  $\gamma$ . Thus, from equation (19) follows that if agents update their value functions more quickly (i.e., the value of  $\gamma$  approaches one), then both trends become more responsive to cycles. From expression (19) it also follows that the “long-run monetary policy response coefficients” affect the persistence of these trends as well as the pass-through of the cycles through the matrix  $Q$ . Thus, to get some insights on the trend and cycle dynamics, and as above noted keeping the assumption that  $\gamma = \tilde{\gamma}$ , the appendix shows that the matrix  $Q$  simplifies to:

$$Q = \frac{1}{\Delta} \begin{pmatrix} 1 - \beta\rho & \sigma(1 - \beta\bar{\phi}_\pi) \\ \kappa & \kappa\sigma + (1 - \rho + \sigma\bar{\phi}_y)\beta \end{pmatrix} \quad (21)$$

where  $\Delta = (1 - \beta\rho)(1 - \rho + \sigma\bar{\phi}_y) + \kappa\sigma(\bar{\phi}_\pi - \rho)$ .

When monetary policy responds more aggressively to trend inflation, then trend inflation becomes less sensitive to movements in cyclical inflation or output. Trend output also becomes less responsive to movements in cyclical output; however, trend output falls more in response to a cyclical increase in inflation for larger values of  $\bar{\phi}_\pi$  assuming  $\bar{\phi}_\pi > 1$ . Similarly, when monetary policy responds more aggressively to trend output, trend output becomes less sensitive to cyclical movements in inflation or output. Trend inflation also becomes less sensitive to cyclical fluctuations in output; however, trend inflation tends to become more responsive to cyclical fluctuations in inflation for larger values of  $\bar{\phi}_y$ .

These results highlight that the FH approach gives rise to a theory through which trend and cycle can be correlated. This idea has been considered in reduced-form econometric analysis since at least since [Nelson and Plosser \(1982\)](#). But, in statistical models, allowing for such a correlation can make identification difficult without stark assumptions (i.e., independence of trend and cycle). In the finite-horizon approach, theoretical restrictions from the model preclude confounding of trend and cycle. Moreover, the model’s trend-cycle decomposition can be directly related to monetary policy and to assumptions about household and firm behavior. In addition, the finite-horizon approach allows one to decompose the cycle and trend into structural shocks. However, it remains an open question how well such an approach can explain aggregate data. This is the key question that we investigate in our empirical analysis.

### 3.2 Dynamic Responses to a Monetary Policy Shock

An important feature of the model is its ability to generate endogenous persistence without any need for habit persistence or the indexation of inflation to past values of inflation. To illustrate this property, we examine the impulse responses to a shock that affects the monetary policy rule. This shock is assumed to follow an AR(1) process:

$$i_t^* = \rho_{i^*} i_{t-1}^* + \epsilon_{i,t} \quad (22)$$

We examine a policy tightening for three different parameterizations. In the first,  $\rho = 1.0$ , which corresponds to the forward-looking, canonical NK model in which the responses of the aggregate and cyclical variables are the same, as the model’s trend corresponds to the nonstochastic steady state. In Figure 1, the canonical NK model’s impulse responses are labelled “Forward”. In the second and third parameterizations of the model, we set  $\rho = 0.5$  which corresponds to 50 percent of households and firms doing their planning within the existing quarter, 25 percent of them doing it in two quarters, and only a small fraction – less than 0.5 percent – of households and firms having a planning horizon of two years or more. The second parameterization, labelled “Large gain” in Figure 1, sets  $\gamma = 0.5$ , which implies that households and firms put a relatively large weight on current observations in updating their value functions. The third parameterization, labelled “Small gain”, is the same as the second one except that  $\gamma = 0.05$ . This value implies that current observations get a relatively small weight in the updating of agents’ value functions.<sup>11</sup>

Figure 1 displays the impulse responses of output,  $y_t$ , inflation,  $\pi_t$ , and the short-term interest rates,  $i_t$  to a unit increase in  $\epsilon_{i,t}$  at date 0. (All variables are expressed in deviation from their values in the nonstochastic steady state.) The first row in the figure corresponds to the responses of the aggregate variables, the second row to the trend responses, and the third row to the cyclical responses. As shown in the first row of the figure, a policy tightening results in an immediate fall in output of a little more than 2 percent and a 15 basis point fall in inflation in the canonical model (green lines). Thereafter, the responses of output and inflation converge back monotonically to their steady state values. This monotonic convergence entirely reflects the persistence of the shock. The middle and lower panels of the figure confirm that in the canonical model, there is no difference between the trends and steady state values of the model so that the aggregate and cyclical responses are the same.

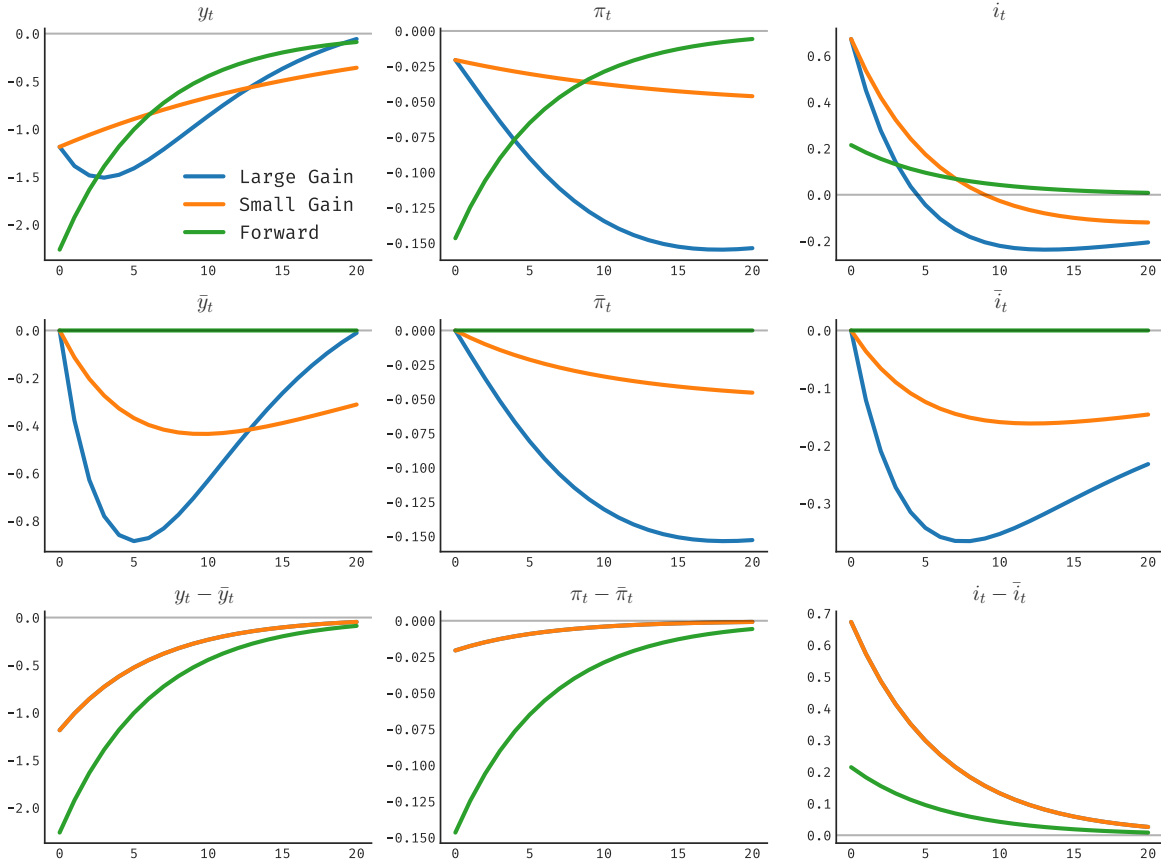
The blue lines, labelled “Large Gain,” in Figure 1 show the impulse responses in the finite horizon model in which agents heavily weigh recent data in updating their value functions. As in the canonical NK model, aggregate output and inflation fall on impact; however, the fall is dampened substantially. Moreover, output and inflation display hump-shaped dynamics despite the lack of indexation or habit persistence in consumption. While output reaches its peak decline after about a year, it takes substantially longer for inflation to reach its peak decline. As shown in the middle panel, these hump-shaped dynamics are driven by the gradual adjustment of the trends. The trend values for output and inflation fall in response to the policy tightening, reflecting that the policy shock persistently lower aggregate output and inflation. For output this return back to trend is relatively quick with a slight overshoot (not shown). However, the inflation trend returns back to its steady state very gradually as agents with finite horizons only come to realize slowly over time that the policy tightening will have a persistent but not permanent effect on inflation.

The orange lines, labelled “Small Gain,” show a similar parameterization except that agents update their value function even more slowly. In this case, the responses of the output and inflation

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<sup>11</sup>For these three cases, we set the remaining parameters as follows:  $\beta = 0.995$ ,  $\sigma = 1$ ,  $\kappa = 0.01$ ,  $\phi_\pi = 1.5$ ,  $\phi_y = \frac{0.5}{4}$ , and  $\rho_{i^*} = 0.85$ .

Figure 1: Impulse Responses to an Unexpected Monetary Tightening



NOTE: The figure shows impulse responses to a monetary policy shock. In the Forward model (red lines), agents have infinite planning horizons ( $\rho = 1.0$ ), and two in the two remaining models, agents have finite planning horizons ( $\rho = 0.5$ ). The first of these models, Large Gain (blue lines), agents learn their value function quickly, ( $\gamma = 0.5$ ); in the second one, Small Gain (green lines), agents learn their value function slowly ( $\gamma = 0.05$ ).

trends is smaller and even more drawn out over time. Because of the dampened response of trend output, the response of aggregate output is no longer hump-shaped, as the aggregate effect is driven primarily by the monotonic cyclical response shown in the bottom left panel. In contrast, the aggregate inflation response is both dampened and more persistent. In sum, the finite horizon model is capable of generating substantial persistence in inflation and hump-shaped output responses following a monetary policy shock. Such dynamics are in line with empirical work examining the effects of monetary policy shocks on the macroeconomy.<sup>12</sup>

<sup>12</sup>See, for instance, [Christiano, Eichenbaum, and Evans \(2005\)](#) and the references therein.



## 4 Estimation

### 4.1 Data and Methodology

We estimate several variants of the model using U.S. data on output growth, inflation, and nominal interest rates from 1966:Q1 through 2007:Q4, a time period for which there were notable changes in trends in inflation and output.<sup>13</sup> The observation equations for the model are:<sup>14</sup>

$$\text{Output Growth}_t = \mu^Q + y_t - y_{t-1} \quad (23)$$

$$\text{Inflation}_t = \pi^A + 4 \cdot \pi_t \quad (24)$$

$$\text{Interest Rate}_t = \pi^A + r^A + 4 \cdot i_t, \quad (25)$$

where  $\pi^A$  and  $r^A$  are parameters governing the model's steady state inflation rate and real rate, respectively. Also,  $\mu^Q$  is the growth rate of output, as we view our model as one that has been detrended from an economy growing at a constant rate,  $\mu^Q$ . Thus, as emphasized earlier, we are using the model to explain low frequency trends in the data but not the average growth rate or inflation rate which are exogenous.

The solution to the system of equations (15) and (19) jointly with these observations equations define the measurement and state transition equations of a linear Gaussian state-space system. The state-space representation of the DSGE model has a likelihood function,  $p(Y|\theta)$ , where  $Y$  is the observed data and  $\theta$  is a vector comprised of the model's structural parameters. We estimate  $\theta$  using a Bayesian approach in which the object of the interest is the posterior distribution of the parameters  $\theta$ . The posterior distribution is calculated by combining the likelihood and prior distribution,  $p(\theta)$ , using Bayes theorem:

$$p(\theta|Y) = \frac{p(Y|\theta)p(\theta)}{p(Y)}.$$

The prior distribution for the model's parameters is generated by a set of independent distributions for each of the structural parameters that are estimated. These distributions are listed in Table 1. For the shocks, we assume they follow AR(1) processes and use relatively uninformative priors regarding the coefficients governing these processes. Specifically, the monetary policy shock follows the AR(1) process given by equation (22) and the processes for the other two shocks are given by:

$$\xi_t = \rho_\xi \xi_{t-1} + \epsilon_{\xi,t} \quad (26)$$

$$y_t^* = \rho_{y^*} y_{t-1}^* + \epsilon_{y^*,t}. \quad (27)$$

The prior for each of the AR(1) coefficients is assumed to be uniform over the unit interval, while

<sup>13</sup>The appendix details the construction of this data.

<sup>14</sup>We reparameterize  $\beta$  to be written in terms in the of the annualized steady-state real interest rate:  $\beta = 1/(1 + r^A/400)$ .

Table 1: Prior Distributions

Parameter	Distribution		
	Type	Par(1)	Par(2)
$r^A$	Gamma	2	1
$\pi^A$	Normal	4	1
$\mu^Q$	Normal	0.5	0.1
$(\rho, \gamma, \tilde{\gamma})$	Uniform	0	1
$\sigma$	Gamma	2	0.5
$\kappa$	Gamma	0.05	0.1
$\phi_\pi$	Gamma	1.5	0.25
$\phi_y$	Gamma	0.25	0.25
$(\sigma_\xi, \sigma_{y^*}, \sigma_{i^*})$	Inv. Gamma	1	4
$(\rho_\xi, \rho_{y^*}, \rho_{i^*})$	Uniform	0	1

NOTE: Par(1) and Par(2) correspond to the mean and standard deviation of the Gamma and Normal distributions and to the upper and lower bounds of the support for the Uniform distribution. For the Inv. Gamma distribution, Par(1) and Par(2) refer to  $s$  and  $\nu$  where  $p(\sigma|\nu, s)$  is proportional to  $\sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$ .

each of the priors for the standard deviations of shocks' is assumed to be an inverse gamma distribution with 4 degrees of freedom.

The priors for the gain parameters,  $\gamma$  and  $\tilde{\gamma}$ , in the household's and firm's learning problems are also assumed to follow uniform distributions over the unit interval. Similarly, we assume that the prior distribution for the parameter governing the length of agents' planning horizons,  $\rho$ , is also a uniform distribution over the unit interval. The prior for  $r^A$  and  $\pi^A$  are chosen to be consistent with a 2% average real interest rate and 4% average rate of inflation. The prior of the slope of the Phillips curve,  $\kappa$ , is consistent with moderate-to-low pass through of output to inflation.<sup>15</sup> The prior for  $\sigma$ , the coefficient associated with degree of intertemporal substitution, follows a Gamma distribution with a mean of 2 and standard deviation of 0.5, and hence encompasses the log preferences frequently used in the literature. The prior distributions of the coefficients of the monetary policy rule,  $\phi_\pi$  and  $\phi_\gamma$ , are consistent with a monetary authority that responds strongly to inflation and moderately to the output gap and encompasses the parameterization in [Taylor \(1993\)](#).

Because we can only characterize the solution to our model numerically, following [Herbst and Schorfheide \(2014\)](#), we use Sequential Monte Carlo techniques to generate draws from the posterior distribution. [Herbst and Schorfheide \(2015\)](#) provide further details on Sequential Monte Carlo and Bayesian estimation of DSGE models more generally. The appendix provides information about the tuning parameters used to estimate the model.

<sup>15</sup>The parameter  $\kappa$  is a reduced form parameter that is related to the fraction of firms that have an opportunity to reset their price,  $1 - \alpha$ , a parameter governing the elasticity of substitution for each price-setter's demand,  $\theta$ , the elasticity of production to labor input,  $\frac{1}{\phi}$ , and the Frisch labor supply elasticity,  $\nu$ . The mean value of our prior for  $\kappa$  is 0.05, which implies an  $\alpha \approx \frac{1}{3}$  with  $\nu = 1$ ,  $\theta = 10$ , and  $\phi = 1.56$ . Thus, the mean of the prior for  $\kappa$  is consistent with an average duration of a firm's price contract that is under one year.

Table 2: Key Parameters of the Estimated Models

Model Type	Parameters		
	Estimated	Fixed	Not identified
Forward	$\phi_\pi, \phi_y$	$\rho = 1$	$\gamma, \tilde{\gamma}, \bar{\phi}_\pi, \bar{\phi}_y$
Stat. Trends	AR(1) trends	$\rho = 1$	$\gamma, \tilde{\gamma}, \bar{\phi}_\pi, \bar{\phi}_y$
FH-baseline	$\rho, \gamma, \phi_\pi, \phi_y$	$\gamma = \tilde{\gamma}, \bar{\phi}_\pi = \phi_\pi, \bar{\phi}_y = \phi_y$	-
FH- $\tilde{\gamma}$	$\rho, \gamma, \tilde{\gamma}, \phi_\pi, \phi_y$	$\bar{\phi}_\pi = \phi_\pi, \bar{\phi}_y = \phi_y$	-
FH- $\bar{\phi}$	$\rho, \gamma, \phi_\pi, \phi_y, \bar{\phi}_\pi, \bar{\phi}_y$	$\gamma = \tilde{\gamma}$	-

NOTE: This table presents the key parameters of the different estimated models.

## 4.2 Models

Table 2 displays the models that we estimate. These models differ in the restrictions on the parameters governing the length of the horizon, the parameters governing how quickly firms and households update their value functions, and the parameters in the reaction function for monetary policy.

The first model, referred to as ‘‘Forward’’ in Table 2, corresponds to the canonical New Keynesian model with three shocks, purely forward looking agents, and a Taylor-type rule for monetary policy. It is consistent with setting  $\rho = 1$ . Because the trends in this model are simply constants, we also consider a version of this model, ‘‘Stat. Trends,’’ which allows for stochastic trends as in [Canova \(2014\)](#) and [Schorfheide \(2013\)](#). Specifically, with  $\rho = 1$ , we augment the model with three more shocks that allow the trends for output, inflation, and the nominal interest rate to evolve exogenously:

$$\bar{y}_t = \rho_{\bar{y}} \bar{y}_{t-1} + \epsilon_{\bar{y},t} \quad (28)$$

$$\bar{\pi}_t = \rho_{\bar{\pi}} \bar{\pi}_{t-1} + \epsilon_{\bar{\pi},t} \quad (29)$$

$$\bar{i}_t = \rho_{\bar{i}} \bar{i}_{t-1} + \epsilon_{\bar{i},t}. \quad (30)$$

The remaining models in Table 2 are all different versions of the FH model. The first, referred to as ‘‘FH-baseline’’, estimates  $\rho$  and  $\gamma$  but assumes that the constant gain parameter,  $\gamma$ , is the same across households and firms. In addition, in this baseline version, the intercept term in the central bank’s reaction function responds to trends in inflation and output in the same manner as it does to short-run cyclical fluctuations (i.e.,  $\bar{\phi}_\pi = \phi_\pi, \bar{\phi}_y = \phi_y$ ). The second variant of the FH model, referred to as ‘‘FH- $\tilde{\gamma}$ ’’, allows for firms and households to learn about their value function at different rates so that  $\gamma$  and  $\tilde{\gamma}$  may differ. The third variant of the FH model, referred to as ‘‘FH- $\bar{\phi}$ ’’, allows for the parameters governing the policy response to trends to differ from those governing the cyclical response of policy.

### 4.3 Results

*Parameter Estimates.* Table 3 displays the means and standard deviations from the posterior distribution of the estimated parameters. The results suggest that incorporating finite horizons into an otherwise canonical NK model is helpful in accounting for movements in U.S. output, inflation and interest rates over the 1966-2007 period. In particular, the estimates of  $\rho$  in the FH versions of the model are all substantially less than one. Such estimates are consistent, but not identical, with the recent evidence in Gabaix (2018), who estimates that the values for discounting future output and inflation are around 0.75. In comparison, these mean estimates shown in Table 3 are closer to 0.5. As discussed earlier, a value of  $\rho = 0.5$  substantially reduces the degree of forward-looking behavior and as a result dampens the responsiveness of output to interest rate changes and inflation to changes in the cyclical position of the economy. For example, using  $\beta = 0.995$ , in the canonical NK model, the effect on current inflation of a (constant) of a 1 percentage point increase in the output gap over eight consecutive quarters is  $\kappa \frac{1-\beta^9}{1-\beta} \bar{y} \approx 9\kappa\bar{y}$ . In contrast, in the FH-baseline model with  $\rho = 0.5$ , this response is given by  $\kappa \frac{1-(\beta\rho)^9}{1-\beta\rho} \bar{y} \approx \kappa\bar{y}$  and is about 9 times smaller.

The estimates also suggest that the slow updating of agents' value functions is helpful in explaining aggregate data. In particular, for all three FH models, the posterior distributions for  $\gamma$  are concentrated at low values, with means around 0.1. For the "FH- $\tilde{\gamma}$ " model, the posterior distribution of  $\tilde{\gamma}$ , with a mean of 0.17, is similarly consistent with slow updating. Thus, households and firms both update their value functions relatively slowly to the new data that they observe, imparting considerable persistence into trend variables. As a result of this sluggishness, the supply shock is much less persistent in the FH versions of the model than in the canonical NK model. In particular, the mean estimate of  $\rho_{y^*}$  is near one in the canonical NK model and close to 0.5 in the FH-baseline model.

Figure 2 provides additional information about the posterior distribution for  $\rho$  and  $\gamma$  derived from the FH- $\bar{\phi}$  model. The grey dots represent draws from the prior distribution while the blue dots represent draws from the posterior distribution. As indicated by the much smaller blue region than the grey region, there is substantial information about the values  $\rho$  and  $\gamma$  in the data. In particular, while the prior contains many draws of  $\rho$  near one, there are essentially zero posterior draws greater than 0.75. This is substantial evidence against models in which  $\rho$  is high including the canonical NK model in which  $\rho = 1$ . The data are also very informative about  $\gamma$  which determines how quickly the finite-horizon households and firms update their value functions. The posterior distribution for  $\gamma$  lies almost entirely between 0.05 and 0.2, which implies that agent's update their value functions very slowly and that trends in inflation, output, and the interest rate are highly persistent.

The estimated coefficients of the monetary policy rule imply that the policy rate is less responsive to cyclical movements in inflation and the output gap in the FH versions of the model than in the canonical NK model. For example, the responsiveness of the policy rate to inflation deviations is about 1.5 in the canonical model and in the Stat. Trends model compared to a value close to 1 in the FH-baseline.

Table 3: Posterior Distributions

	Forward		Stat. Trends		FH-baseline		FH- $\bar{\phi}$		FH- $\bar{\gamma}$	
	mean	std.	mean	std.	mean	std.	mean	std.	mean	std.
$r^A$	2.36	0.47	1.95	0.81	2.51	0.38	2.39	0.31	2.52	0.48
$\pi^A$	4.01	0.88	4.16	0.75	3.95	0.99	3.83	0.93	3.97	1.01
$\mu^Q$	0.42	0.02	0.43	0.03	0.45	0.01	0.45	0.02	0.45	0.01
$\rho$					0.5	0.13	0.42	0.13	0.46	0.13
$\gamma$					0.13	0.03	0.11	0.02	0.09	0.05
$\bar{\gamma}$									0.17	0.06
$\sigma$	1.38	0.39	1.73	0.47	3.48	0.60	3.75	0.62	3.59	0.59
$\kappa$	0.01	0.02	0.00	0.00	0.04	0.01	0.04	0.01	0.04	0.02
$\phi_\pi$	1.54	0.24	1.49	0.21	1.08	0.13	0.96	0.15	1.10	0.14
$\phi_y$	0.92	0.17	0.86	0.19	0.78	0.16	0.73	0.15	0.77	0.16
$\bar{\phi}_\pi$							2.03	0.26		
$\bar{\phi}_y$							0.06	0.06		
$\rho_\xi$	0.76	0.04	0.73	0.09	0.97	0.02	0.97	0.02	0.95	0.04
$\rho_{y^*}$	0.96	0.02	0.75	0.33	0.53	0.09	0.59	0.08	0.52	0.11
$\rho_{i^*}$	0.95	0.03	0.97	0.02	0.97	0.01	0.97	0.02	0.97	0.01
$\sigma_\xi$	2.21	0.56	1.22	0.29	2.10	0.37	1.96	0.31	1.98	0.31
$\sigma_{y^*}$	1.50	0.53	1.24	0.38	5.66	1.88	4.97	1.09	5.21	1.43
$\sigma_{i^*}$	0.82	0.13	0.73	0.15	0.67	0.12	0.58	0.11	0.66	0.12
Log MDD	-758.20	1.22	-718.63	2.16	-727.01	0.94	-716.54	1.34	-728.27	1.19

Another important feature of the estimated policy rule is that the data prefers rule coefficients that differ significantly in the short run from those in the long run. In the FH- $\bar{\phi}$  version of the model, the coefficient on trend inflation deviations is near 2 while the coefficient on trend output deviations is close to zero. Hence, the monetary policy rule responds more aggressively to stabilize deviations of trend inflation from the steady state inflation rate than it does to short-run inflation deviations from trend. In addition, policy responds aggressively to short-run deviations of output from trend but very little to the deviation of trend output from steady state.

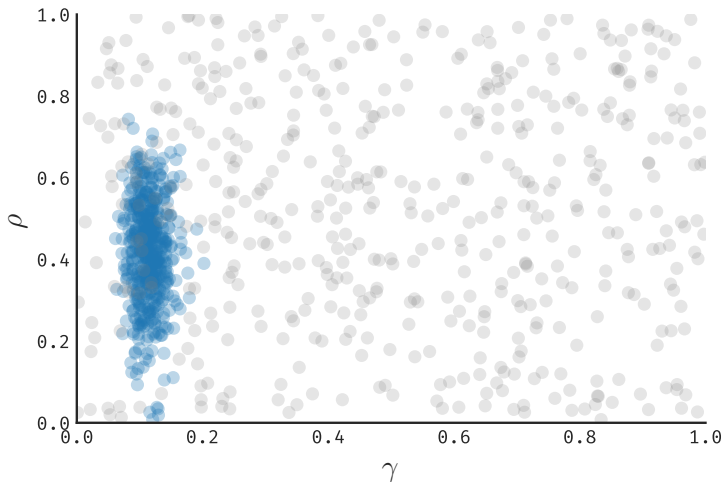
*Model Fit.* The last row of Table 3 shows, for each model, an estimate of the log marginal data density, defined as:

$$\log p(Y) = \log \left( \int p(Y|\theta)p(\theta)d\theta \right).$$

This quantity provides a measure of overall model fit, and an estimate of it is computed as a by-product of the Sequential Monte Carlo algorithm.<sup>16</sup> The data favors the FH- $\bar{\phi}$  version of the model which allows for monetary policy to respond more aggressively to deviations in trend inflation than to short-run deviations of inflation from trend. This model fits substantially better than the canonical NK model. More interestingly, the model also fits moderately better than incorporating additional shocks into the canonical model to allow for stochastic trends in inflation, output, and interest rates. Overall, the estimates suggest that allowing for agents with finite-horizons, slow

<sup>16</sup>The standard deviation of the Log MDD is computed across 10 runs of the algorithm.

Figure 2: Joint Posterior Distribution of Parameters  $\rho$  and  $\gamma$



NOTE: The grey dots represent draws from the prior distribution of  $(\rho, \gamma)$  while the blue dots represent draws from the posterior distribution of  $(\rho, \gamma)$  from the FH- $\bar{\phi}$  model.

learning about the observed trends, and an aggressive policy response to trend inflation are all important in accounting for movements in inflation, output, and interest rates.

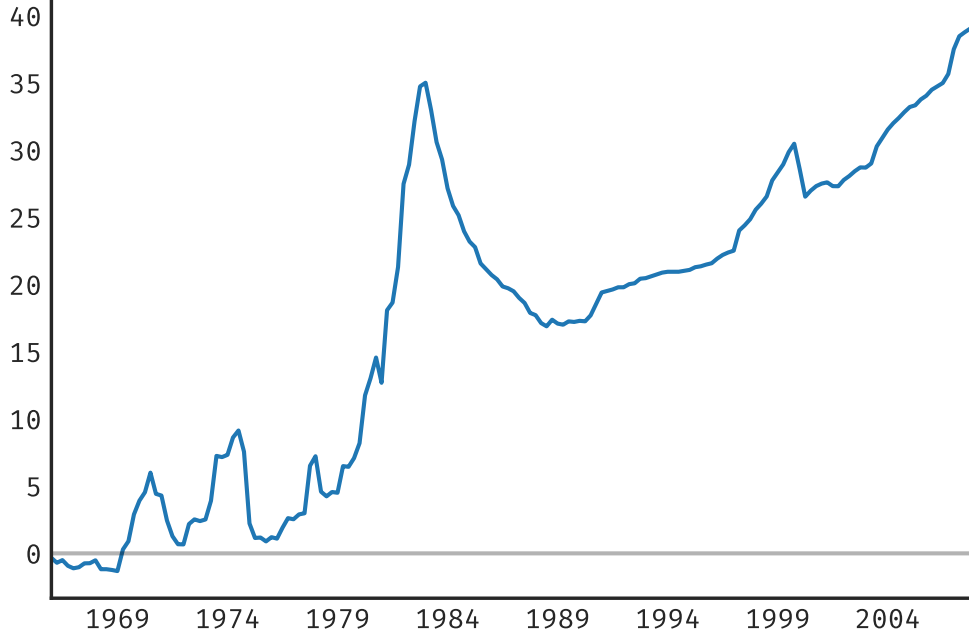
Figure 3 compares the fit of the FH model relative to the canonical NK model over an expanding sample. Specifically, the figure plots

$$\Delta_t = \log \hat{p}_{\text{FH}-\bar{\phi}}(Y_{1:t}) - \log \hat{p}_{\text{Forward}}(Y_{1:t}), \quad (31)$$

where  $Y_{1:t}$  is a matrix that includes the observables through period  $t$  and  $\log \hat{p}_{\mathcal{M}}(Y_{1:t})$  is an estimate of the log marginal data density for model  $\mathcal{M}$  for the subsample of  $Y$  that ends in period  $t$ . Thus,  $\Delta_t$  measures the cumulative difference in the mean estimates of the log marginal data density for the FH- $\bar{\phi}$  from the canonical NK model. The figure shows that the data strongly prefers the FH- $\bar{\phi}$  beginning in the late 1970s and early 1980s. For the canonical NK model, this period is difficult to rationalize, since it must capture the upward inflation trend in the 1970s and large deviations of inflation in the 1980s through large and persistent shocks. In contrast, the FH- $\bar{\phi}$  model embeds persistence into trend inflation that makes it easier to fit the Great Inflation episode. Although the relative fit of the canonical NK model improves somewhat during the Volker disinflation, as inflation moves back toward the model's mean estimate for  $\pi^A$  of 4 percent, it continues to fit much worse than the FH- $\bar{\phi}$  for the remainder of the sample. This better fit of the FH- $\bar{\phi}$  model reflects that this model does a relatively good job capturing the secular decline in inflation, as inflation moves and remains well below 4 percent over the latter part of the sample.

*Monetary Policy Shocks in the Estimated Model.* As discussed earlier, empirical evidence from

Figure 3: Difference in Log MDD over time



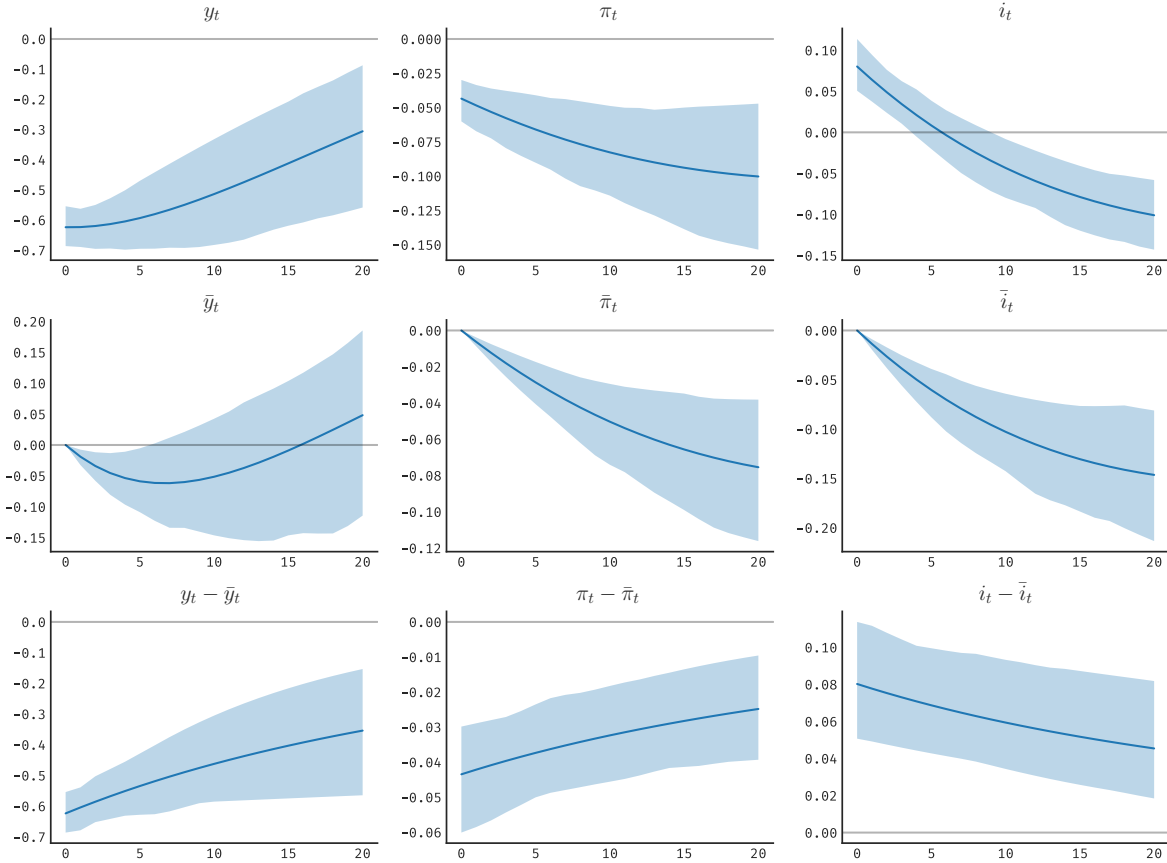
NOTE Figure displays  $\Delta_t$  (defined in (31)).

the VAR literature has emphasized that following a monetary policy shock, there is considerable persistence in the price response and a delayed response in output. Figure 4 plots the 90-percent pointwise credible bands for impulse responses of output, inflation, and the short-term interest rate to a one standard deviation increase in  $\epsilon_{i,t}$  from the FH- $\bar{\phi}$  model. There is a persistent fall in output following a tightening in monetary policy with the decline in output after one year on par with the initial fall. This response in part reflects the hump shaped pattern in trend output, which falls slowly over the next year or so before recovering. As shown earlier, the responses from the estimated model for inflation are highly persistent. Inflation only drops slightly on impact and its response grows over time as agents revise down their estimates of the trend. Overall, however, its response is small.

*Trend-cycle decomposition of inflation, output and the policy rate.* Figure 5 decomposes observed inflation into its trend and cyclical components. The top panel displays the filtered estimates from the FH- $\bar{\phi}$  model of trend inflation in the top panel. Trend inflation, according to the model, rose sharply during the 1970s, declined during the 1980s, and then remained relatively constant from 1990 to 2007. The middle panel shows that the model’s measure of the deviation of inflation



Figure 4: Impulse Responses to a Monetary Policy Tightening



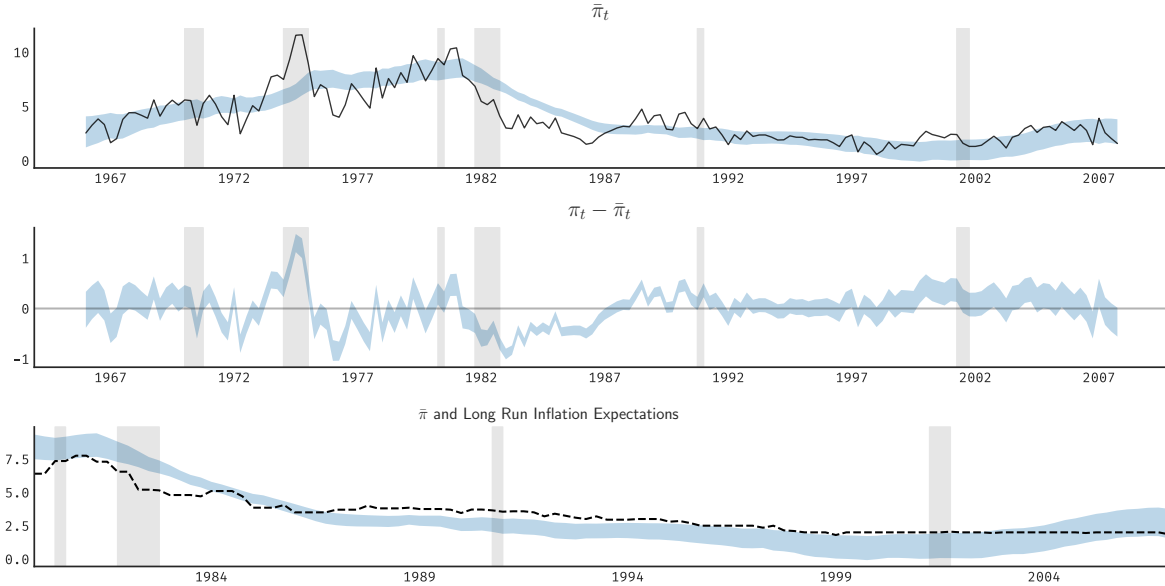
NOTE: This figure plots the posterior mean and the 90-percent pointwise credible bands for impulse responses of model variables to a one standard deviation increase in  $\epsilon_{i,t}$  for the FH- $\bar{\phi}$  model using 250 draws from the posterior distribution.

from trend displays little persistence with the possible exception of the early 1980s when inflation remained below trend for a couple of years. Moreover, as the middle panel suggests, the model's estimate of  $\pi_t - \bar{\pi}_t$  implies that the volatility of inflation relative to trend declined during the period of the Great Moderation. The bottom panel of Figure 5 compares the FH- $\bar{\phi}$  model's trend inflation estimates to the median of 10-year average inflation expectations from the Survey of Professional Forecasters.<sup>17</sup> Although the model uses the GDP deflator to compute trend inflation and the survey-based measure is for the CPI, the two series display a similar pattern: both measures fall sharply during the Volcker disinflation and then stabilized in the 1990s at a level well below their respective measures in the early 1980s.

The top panel of Figure 6 displays the smoothed estimates from the FH- $\bar{\phi}$  of the trend interest

<sup>17</sup>This variable from the Survey of Professional Forecasters is available starting in 1991. See the appendix for additional details about this variable.

Figure 5: Trend-Cycle Decomposition: Inflation

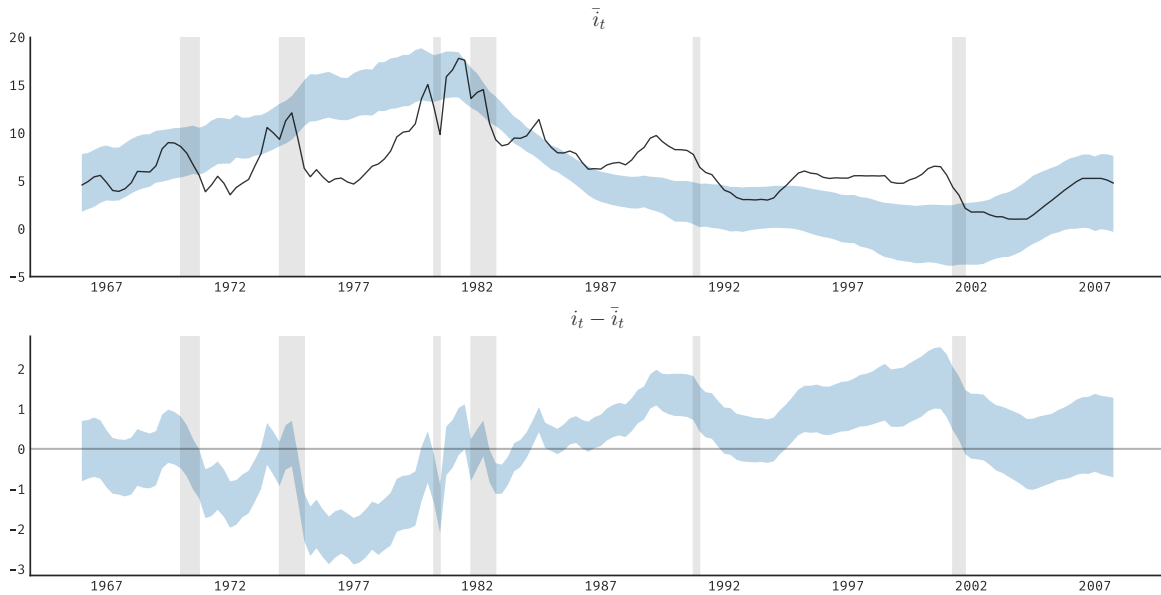


NOTE: The top panel of this figure shows the time series of 90 percent pointwise credible interval for the smoothed mean of  $\bar{\pi}_t$ , annualized and adjusted by  $\pi^A$  (shaded region), as well as observed inflation (solid line.) The middle panel shows the time series of 90 percent pointwise credible interval for the smoothed mean of  $\pi_t - \bar{\pi}_t$  (shaded region). The bottom panel shows the time series of 90 percent pointwise credible interval for the smoothed mean of  $\bar{\pi}_t$ , annualized and adjusted by  $\pi^A$  (shaded region) along with the SPF long run inflation expectations (dashed line).

rate. The trend interest rate follows the same pattern as the model's trend inflation series: rising substantially in the 1970s, falling sharply in the 1980s, and then recovering in the 1990s. The fact that the movements in the trend interest rate is so similar to those for trend inflation in the FH- $\bar{\phi}$  model is not too surprising, since the estimates of that model imply that the trend interest rate is driven almost entirely by trend inflation rather than the trend in output. The middle panel displays FH- $\bar{\phi}$  model estimates of the deviation of the interest rate from trend. The estimates suggest that monetary policy responded by cutting rates aggressively well below trend during the recessions in late 1960s and mid-1970s. In both the recessions of 1981-82 and in 2001,  $i_t - \bar{i}_t$  also fell but from relatively elevated levels.

The top panel of Figure 7 displays the smoothed estimates of the output gap, measured as the deviation of output relative to trend from the FH- $\bar{\phi}$  model. As shown there, the model's estimate of the output gap falls sharply during NBER recession dates. For example, in both of the recessions in the mid-1970s and in 1981-82, the estimate of  $y_t - \bar{y}_t$  falls more than 2 percentage points. In contrast, as shown in the middle panel, the model's estimate of the trend moves much less during NBER recessions. Trend output, for instance, declines somewhat during the severe recession in the mid-1970s but this decline is small relative to the fall in the model's cyclical measure for output. In addition, the level of trend output is unchanged or even increases a bit during other NBER

Figure 6: Trend-Cycle Decomposition: Short-term Interest Rate



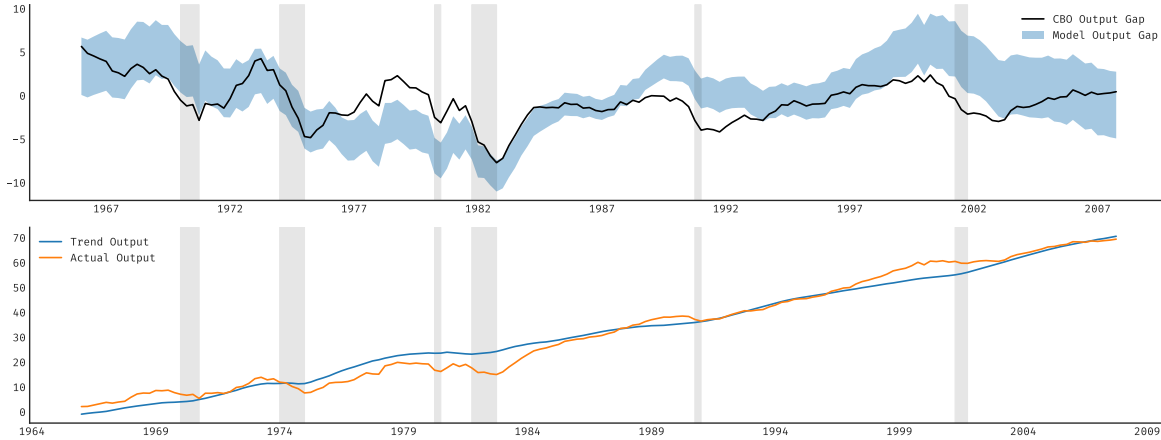
NOTE: The top panel of this figure shows the time series of 90 percent pointwise credible interval for the smoothed mean of  $\bar{i}_t$ , annualized and adjusted by  $\pi^A + r^A$  (shaded region), as well as the observed federal funds rate (solid line.) The bottom panel shows the time series of 90 percent pointwise credible interval for the smoothed mean of  $i_t - \bar{i}_t$  (shaded region).

recessions. The bottom panel of the figure compares the smoothed estimates of the output gap to the output gap measured published by the CBO. The model's estimate of the output gap and the CBO measure have a correlation of about 0.5. The two measures differ notably in terms of how they saw the cyclical position of the economy in the mid to late 1970s and in the late 1990s. While the CBO measure saw a significant improvement in the cyclical position of the economy following the recessions at the end of the 1960s and in the mid-1970s, the model-based measure shows little improvement following those recessions. In addition, the model-based measure of output is well above trend in the late 1970s, while the CBO measure indicates only a modest degree of cyclical pressure.

## 5 Comparison with other Behavioral's NK Models

So far, we have shown how the finite-horizon model can do a better job in accounting for inflation and output dynamics over the Great Inflation and Volcker disinflation periods relative to a NK model that incorporates stochastic trends. It is also interesting to compare the finite-horizon model's performance to other ways of incorporating behavioral features into the NK model. In addition, we compare the model's performance to the hybrid NK model which includes habit persistence and inflation indexation in order to generate persistent movements in output and inflation

Figure 7: Trend-Cycle Decomposition: Output



NOTE: The top panel of the figure shows the level of actual output (orange line) as well as the smoothed mean estimates of the trend level of output (blue line) for the FH- $\bar{\phi}$  model. The bottom panel shows the time series of 90 percent pointwise bands of the cyclical position of output,  $y_t - \bar{y}_t$ , for the FH- $\bar{\phi}$  model, as well as the CBO estimate of the output gap.

in line with the observed data.

The model with finite horizons is closely related to two recent extensions of the NK model. The first is discussed in [Gabaix \(2018\)](#), who departs from rational expectations since agents have distorted beliefs in forecasting variables. [Angeletos and Lian \(2018\)](#) also extend the NK model so that strategic interactions between agents affect their expectations of future variables. Though the microfoundations differ from the finite-horizon approach discussed here, these recent extensions give rise to similar expressions characterizing linearized aggregate dynamics. In [Angeletos and Lian \(2018\)](#), the linearized expressions for output and inflation are given by:

$$y_t = \rho E_t y_{t+1} - \sigma(i_t - \lambda E_t \pi_{t+1} - r_t^n) \quad (32)$$

$$\pi_t = \beta \rho_f E_t \pi_{t+1} + \kappa y_t + u_t \quad (33)$$

where the parameters  $\rho$ ,  $\lambda$ , and  $\rho_f \in [0, 1]$ . The expressions determining aggregate output and inflation in [Gabaix \(2018\)](#) are very similar except that  $\lambda = 1$ .

The expressions (32) and (33) are similar to those determining aggregate inflation and output in the finite-horizon approach; however, in the finite-horizon model,  $\rho$ ,  $\rho_f$ , and  $\lambda$  are constrained to be the same. A more important difference is that the variables in the finite-horizon model are expressed in deviation from trends which are determined endogenously as agents update their value functions. In contrast, this feature is absent from [Angeletos and Lian \(2018\)](#) and the variables are expressed as a deviation from their nonstochastic steady state.

Table 4 compares our measure of model fit, the log marginal data density, for the [Angeletos and](#)

Lian (2018) to the alternative estimated versions of the finite horizon model discussed earlier.<sup>18</sup> The log marginal data density is about 5 points higher in the version of the finite horizon model in which the monetary policy reaction function is the same in the short and long run (labelled "FH") and about 13 points higher when the policy reaction function differs in the long and short run (labelled FH- $\bar{\phi}$ ). Accordingly, the fit of the finite-horizon models is better than the model in Angeletos and Lian (2018) and when the policy reaction function is allowed to differ in the short and long run in the finite-horizon model, the fit is substantially better. This improved fit reflects the endogenous persistence the finite-horizon approach can generate through slow moving trends for output, inflation, and interest rates. Similar results would apply to the model in Gabaix (2018), since the aggregate dynamics of that model (up to a log-linear approximation) are a special case of Angeletos and Lian (2018) with  $\lambda = 1$ .

Because the endogenous persistence generated by household and firms' learning about their value functions is an important feature of the FH approach in accounting for aggregate data, it is interesting to compare its fit with a hybrid NK model. This model introduces persistence into output and inflation by introducing habit formation in the households' preferences and indexation to past inflation of firm's Calvo price contracts and has been used extensively in empirical applications in the literature.<sup>19</sup> In the hybrid NK model, the log-linear aggregate dynamics for output and inflation are given by:

$$\begin{aligned}
(1 + 2\alpha)y_t &= \alpha y_{t-1} + (1 + \alpha)E_t y_{t+1} - E_t [i_t - \pi_{t+1} - \xi_t] \\
[1 + \beta(1 - a)]\pi_t &= (1 - a)\pi_{t-1} + \beta E_t \pi_{t+1} \\
&\quad + \kappa(1 + \alpha)\hat{y}_t - \kappa\alpha\hat{y}_{t-1} + y_t^*
\end{aligned} \tag{34}$$

where  $\alpha = \frac{\nu}{1-\nu}$ , and  $\nu$  is the habit-formation parameter in the households' preferences,  $\beta$  is the discount factor, and  $1 - a$  is the indexation to past inflation of the Calvo's price contracts of firms.

Table 4 shows that the three versions of the finite-horizon model that we estimate all fit the observed dynamics of output, inflation, and the interest rate better than the hybrid NK model. This better fit reflects both the endogenous persistence generated by agents' learning about their value functions as well as the reduced degree of forward-looking behavior associated with  $\rho < 1$ . Overall, the results in this section suggest the finite-horizon approach with agents' learning about their value functions is a parsimonious and fruitful way to fit movements in longer-run trends and aggregate business cycle dynamics.

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<sup>18</sup>The Appendix shows the posterior distributions of the parameters for the Angeletos and Lian (2018) model as well as for the hybrid NK model.

<sup>19</sup>The underlying preferences for the households are  $\log [C_t - \nu C_{t-1}^a] - H_t$ , where  $H(t)$  are hours worked and the parameter  $\nu$  captures the presence of external habits reflecting the influence of "aggregate" past consumption on current utility. Firms set prices in a staggered way (a la Calvo) and price contracts are indexed to past aggregate inflation. The indexation parameter is  $1 - a$ .

Table 4: Ranking Overall Fit of Alternative Models

	Log MDD	
	Mean	Std.
FH- $\phi$	-716.54	1.34
Stat. Trends	-718.63	2.16
FH-baseline	-727.01	0.94
FH- $\tilde{\gamma}$	-728.27	1.19
Angeletos-Lian	-731.00	0.95
Hybrid NK	-734.24	1.55
Forward	-758.20	1.22

Means and standard deviations are over 10 runs of each algorithm.

## 6 Conclusion

Researchers recently retreat from very forward looking rational agents and incorporate some behavioral elements, based on human judgement and cognitive abilities obtained from experimental studies, into the New Keynesian (NK) model. Among the many challenges faced by these new models, a key one is to what extent these models offer the researcher enough flexibility to describe the persistence of aggregate data, and hence its usefulness for policy analysis.

To achieve this objective, we use the theoretical underpinnings of [Woodford \(2018\)](#) in which agents make decisions on the basis of a finite-horizon planning. Households and firms construct the optimal short-horizon plan by applying backward induction over a finite set of periods given some value function assigned to beliefs about the terminal states at the final planning period. These value functions are intended to represent values that decision makers have learned by averaging their past experience with different states, and a sufficient amount of experience with an environment that may persistently differs from the steady state.

Overall, the results suggest that the finite-horizon approach with agents' learning about their value functions is a parsimonious and fruitful way to understand aggregate business cycle dynamics. First, we reject parameterizations of the purely forward looking, canonical model in favor of versions in which at substantial fraction of agents have finite horizons. Second, the observed persistence in aggregate data is informative about how quickly finite-horizon households and firms update their value functions. Our evidence is consistent with agents updating their value functions very slowly. This slow adjustment explains the observed medium-run persistent (trends) in inflation, output, and the interest rate since late 1960s in the U.S. Finally, our results also shed new light on understanding how monetary policy and macroeconomic persistence are intertwined.

A couple of extensions of the current work deserve further research. First, our paper provides estimates of important parameters of the model that can be used to study in depth the rich individual heterogeneity underlying the (aggregate) model. For instance, in line with the recent attempt to study the microeconomic heterogeneity of macro NK models ([Kaplan, Moll, and Violante \(2018\)](#)),

it will be interesting to study the cross-sectional properties of this model regarding income, wealth, and inflation expectations across agents with different planning horizons. Second, the model is sufficiently streamlined so it abstracted from investment dynamics. Incorporating capital accumulation in the context of short-term planning decisions will allow to study the implications of this type of models for aggregate investment and the price of capital jointly with other macroeconomic variables.



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# Appendices

## A Model Dynamics

In this section of the appendix, we provide some of the details that help characterize the dynamics of the finite-horizon model.

### A.1 The Cycle and the Taylor Principle

The system determining the cycle is:

$$\tilde{x}_t = \rho M \cdot E_t[\tilde{x}_{t+1}] + N \cdot u_t, \quad (\text{A-1})$$

where the matrices  $M = \frac{1}{\delta} \begin{pmatrix} 1 & \sigma(1 - \beta\phi_\pi) \\ \kappa & \kappa\sigma + \beta(1 + \sigma\phi_y) \end{pmatrix}$  and  $N = \frac{1}{\delta} \begin{pmatrix} -\sigma & -\sigma\kappa\phi_\pi \\ -\kappa\sigma & \kappa(1 + \sigma\phi_y) \end{pmatrix}$ , with  $\delta = 1 + \sigma(\phi_y + \kappa\sigma_\pi)$ . To determine the Taylor principle for the FH model, rewrite the system (A-1) as

$$E_t[\tilde{x}_{t+1}] = A[\tilde{x}_t] + Bu_t,$$

where the relevant matrix  $A$  is given by

$$A = \begin{pmatrix} (\beta\rho)^{-1} & -\kappa(\beta\rho)^{-1} \\ \sigma(\phi_\pi - \beta^{-1}) & 1 + \sigma(\phi_y + \kappa\beta^{-1}) \end{pmatrix}.$$

The equilibrium is determinate if and only if the matrix  $A$  has both eigenvalues outside the unit circle (i.e., with modulus larger than one). Invoking proposition C.1 in [Woodford \(2003\)](#), this condition is satisfied if and only if

$$\det(A) - \text{tr}(A) > -1.$$

This condition implies:

$$\left( \frac{1 - \beta\rho}{\kappa} \right) \phi_y + \phi_\pi > \rho.$$

### A.2 Trend-Cycle Decomposition

In this section, we report the matrices that determine the evolution of the model's trends. The evolution of  $v_t$  and  $\tilde{v}_t$  is given by:

$$V_{t+1} = (I - \Gamma)V_t + \Gamma\Phi x_t, \quad (\text{A-2})$$

where  $V_t' = (v_t \quad \tilde{v}_t)$ , and the matrices  $\Gamma = \begin{pmatrix} \gamma & 0 \\ 0 & \tilde{\gamma} \end{pmatrix}$  and  $\Phi = \begin{pmatrix} 1 & \sigma \\ 0 & \frac{1}{(1-\alpha)} \end{pmatrix}$ . The trends can be written in terms of  $V_t$  as:

$$\bar{x}_t = (1 - \rho)\Theta V_t, \quad (\text{A-3})$$

where the matrix of coefficients  $\Theta = \frac{1}{\Delta} \begin{pmatrix} 1 - \beta\rho & -\sigma(\bar{\phi}_\pi - \rho)(1 - \alpha)\beta \\ \kappa & (1 - \rho + \sigma\bar{\phi}_y)(1 - \alpha)\beta \end{pmatrix}$  and  $\Delta = (1 - \beta\rho)(1 - \rho + \sigma\bar{\phi}_y) + \kappa\sigma(\bar{\phi}_\pi - \rho)$ .

Combining expression (A-2) with expression (A-3) yields:

$$\bar{x}_t = \Lambda\bar{x}_{t-1} + (1 - \rho)\gamma Qx_{t-1},$$

where  $\Lambda = \Theta(I - \Gamma)\Theta^{-1}$  and  $(1 - \rho)\gamma Q = \Theta\Gamma\Phi$ . After some algebra these matrices can be written as:

$$\Lambda = \frac{1}{\Delta} \begin{pmatrix} (1 - \gamma)(1 - \beta\rho)(1 - \rho + \sigma\bar{\phi}_y) + (1 - \tilde{\gamma})\frac{(\bar{\phi}_\pi - \rho)}{(\sigma\kappa)^{-1}} & \sigma(1 - \beta\rho)(\bar{\phi}_\pi - \rho)(\tilde{\gamma} - \gamma) \\ (\tilde{\gamma} - \gamma)\kappa(1 - \rho + \sigma\bar{\phi}_y) & (1 - \tilde{\gamma})(1 - \beta\rho)(1 - \rho + \sigma\bar{\phi}_y) + (1 - \gamma)\frac{(\bar{\phi}_\pi - \rho)}{(\sigma\kappa)^{-1}} \end{pmatrix}$$

$$Q = \frac{1}{\Delta} \begin{pmatrix} (1 - \beta\rho) & \sigma(1 - \beta\rho) - \frac{\tilde{\gamma}}{\gamma}\sigma(\bar{\phi}_\pi - \rho)\beta \\ \kappa & \kappa\sigma + \frac{\tilde{\gamma}}{\gamma}(1 - \rho + \sigma\bar{\phi}_y)\beta \end{pmatrix}.$$

When  $\gamma = \tilde{\gamma}$ , the system simplifies to:

$$\bar{x}_t = (1 - \gamma)\bar{x}_{t-1} + (1 - \rho)\gamma Q x_{t-1},$$

with  $Q = \frac{1}{\Delta} \begin{pmatrix} 1 - \beta\rho & \sigma(1 - \beta\bar{\phi}_\pi) \\ \kappa & \kappa\sigma + (1 - \rho + \sigma\bar{\phi}_y)\beta \end{pmatrix}$ . Note that in this case the feedback of  $\bar{x}_t$  on its lag can be characterized by the scalar,  $1 - \gamma$ , and that  $Q$  is independent of  $\gamma$ . Finally,  $Q$  can be simplified further if  $\bar{\phi}_y = 0$ :  $Q = \frac{1}{\Delta} \begin{pmatrix} 1 - \beta\rho & \sigma(1 - \beta\bar{\phi}_\pi) \\ \kappa & \kappa\sigma + (1 - \rho)\beta \end{pmatrix}$ , with  $\Delta = (1 - \beta\rho)(1 - \rho) + \kappa\sigma(\bar{\phi}_\pi - \rho) > 0$  if  $\bar{\phi}_\pi > \rho$ .

## B Data

The data used in the estimation is constructed as follows.

1. **Per Capita Real Output Growth.** Take the level of real gross domestic product, (FRED mnemonic “GDPC1”), call it  $GDP_t$ . Take the quarterly average of the Civilian Non-institutional Population (FRED mnemonic “CNP16OV” / BLS series “LNS10000000”), call it  $POP_t$ . Then,

$$\begin{aligned} & \text{Per Capita Real Output Growth} \\ & = 100 \left[ \ln \left( \frac{GDP_t}{POP_t} \right) - \ln \left( \frac{GDP_{t-1}}{POP_{t-1}} \right) \right]. \end{aligned}$$

2. **Annualized Inflation.** Take the GDP deflator, (FRED mnemonic “GDPDEF”), call it  $PGDP_t$ . Then,

$$\text{Annualized Inflation} = 400 \ln \left( \frac{PGDP_t}{PGDP_{t-1}} \right).$$

3. **Federal Funds Rate.** Take the effective federal funds rate (FRED mnemonic “FEDFUNDS”), call it  $FFR_t$ . Then,

$$\text{Federal Funds Rate} = FFR_t.$$

The figures in the paper include two additional series, the CBO estimate of the Output Gap and longer-run inflation expectations. These data are constructed as follows.

1. **CBO Output Gap.** The CBO’s estimate of the level of Potential GDP, (FRED mnemonic “GDPPOT”), call it  $POT_t$ .

$$\text{CBO Output Gap}_t = 100 \ln \left( \frac{GDP_t}{POT_t} \right).$$

2. **Longer-run Inflation Expectations.** This data comes by merging the SPF 10-year average inflation expectations (available starting in 1991Q4), with a similar measure from Blue Chip Economic Indicators from (1979Q4-1991Q2), call it “ $\pi_t^{10}$ ”. The data construction follows [Del Negro, Giannoni, and Schorfheide \(2015\)](#). Since these are CPI-based measures, we subtract fifty basis points to account for persistent differences between CPI and GDP-based measures of inflation.

$$\text{Longer-run Inflation Expectations}_t = \pi_t^{10} - 0.5.$$

## C Computational Details

Our estimation follows [Herbst and Schorfheide \(2014\)](#) with the following hyperparameters:  $N_{part} = 4000$ ,  $N_{\phi} = 500$ ,  $\lambda = 2.1$ ,  $N_{blocks} = 1$ ,  $N_{intmh} = 1$ . We run each sampler 10 times, and pool the draws from the runs, yielding a posterior distribution with 40,000 draws.

## D Additional Tables

Table [A-1](#) displays moments of the prior and posterior distribution for the parameters associated with the exogenous processes for the Stat. Trend model. Note that the prior distribution is informative; the distribution is consistent with the view that these trends are very persistent and that the magnitude of their innovations are small relative to the shocks of the DSGE model.

Table A-1: Posterior Distribution of Stat. Trends Model

Parameter	Prior		Posterior	
	Mean	Std.	Mean	Std.
$\rho_{\bar{i}}$	0.95	0.05	0.96	0.03
$\rho_{\bar{y}}$	0.95	0.05	0.97	0.03
$\rho_{\bar{\pi}}$	0.95	0.05	0.92	0.03
$\sigma_{\bar{i}}$	0.12	0.04	0.14	0.05
$\sigma_{\bar{y}}$	0.12	0.04	0.11	0.03
$\sigma_{\bar{\pi}}$	0.12	0.04	0.23	0.03

Table [A-2](#) displays moments of the posterior distribution of the [Angeletos and Lian \(2018\)](#) model, the Hybrid New Keynesian model, and the FH- $\bar{\phi}$ .

Table A-2: Posterior Distributions: Alternative Models

	Angeletos-Lian		Hybrid NK		FH- $\phi$	
	mean	std	mean	std	mean	std
$r^A$	1.96	0.80	2.19	0.51	2.39	0.31
$\pi^A$	3.86	0.89	3.97	0.87	3.83	0.93
$\mu^Q$	0.41	0.02	0.44	0.03	0.45	0.02
$\rho$	0.79	0.12			0.42	0.13
$\rho_f$	0.09	0.07				
$\gamma$					0.11	0.02
$\iota_b$			0.05	0.02		
$\iota_f$			0.98	0.02		
$\kappa$	0.04	0.05	0.01	0.00	0.04	0.01
$\sigma$	1.94	0.51	1.71	0.42	3.75	0.62
$\lambda$	0.79	0.28				
$\nu$			0.88	0.04		
$\phi_\pi$	1.41	0.22	1.98	0.24	0.96	0.15
$\bar{\phi}_\pi$					2.03	0.26
$\phi_y$	0.56	0.18	0.13	0.04	0.73	0.15
$\bar{\phi}_y$					0.06	0.06
$\rho_{i^*}$	0.98	0.01	0.94	0.02	0.97	0.02
$\rho_\xi$	0.87	0.05	0.40	0.09	0.97	0.02
$\rho_{y^*}$	0.97	0.02	0.96	0.01	0.59	0.08
$\sigma_{i^*}$	0.58	0.11	0.55	0.06	0.58	0.11
$\sigma_\xi$	0.36	0.04	3.06	0.78	1.96	0.31
$\sigma_{y^*}$	1.50	0.56	0.99	0.28	4.97	1.09