Correcting the Analytical Determinacy Boundary in Gust et al. (2022)

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June 6, 2025

Abstract. This note corrects a sign and ordering error in the matrix inversion appearing in Section II.A of "Short-term Planning, Macroeconomic Persistence, and Monetary Policy" (*American Economic Journal: Macroeconomics* Vol. 14, No. 4, October 2022, pp. 174–209). The misprint affects the analytical expression for the determinacy boundary (equation 20), but *not* the numerical results reported in the paper: all equilibrium classifications in the main text were obtained with GENSYS, so the posterior estimates and impulse responses remain unchanged. We derive the correct condition, $(1-\rho+\sigma\varphi_y)(1-\beta\rho)+\kappa\sigma(\varphi_\pi-\rho)>0$. We thank Emi Nakamura, Venance Riblier, and Jón Steinsson for bringing the error and correction to our attention.

Begin with

$$\tilde{x}_t = \rho M \mathbb{E}_t[\tilde{x}_{t+1}] + N u_t, \qquad \tilde{x}_t = \begin{bmatrix} \tilde{y}_t \\ \tilde{\pi}_t \end{bmatrix}.$$

where

$$M = \frac{1}{\delta}\begin{bmatrix} 1 & \sigma(1-\beta\varphi_\pi) \\ \kappa & \kappa\sigma+\beta(1+\sigma\varphi_y) \end{bmatrix}, \qquad \delta = 1+\sigma(\varphi_y+\kappa\sigma\varphi_\pi).$$

The original version incorrectly derives the inverse $A=\rho^{-1}M^{-1}$. The correct version is:

$$A = \rho^{-1}M^{-1} = \frac{1}{\beta\rho}\begin{bmatrix} \beta(1+\sigma\varphi_y)+\kappa\sigma & \sigma(\beta\varphi_\pi-1) \\ -\kappa & 1 \end{bmatrix}.$$

We have:

$$\det A = \frac{1+\sigma\varphi_y+\kappa\sigma\varphi_\pi}{\beta\rho^2}, \qquad \operatorname{tr} A = \frac{\beta(1+\sigma\varphi_y)+\kappa\sigma+1}{\beta\rho}.$$

The conditions for determinacy are:

(J1) $\det A > 1 \iff \kappa\sigma\varphi_\pi + \sigma\varphi_y + 1 > \beta\rho^2,$

(J2) $\det A - \operatorname{tr} A + 1 > 0 \iff (1-\rho+\sigma\varphi_y)(1-\beta\rho)+\kappa\sigma(\varphi_\pi-\rho) > 0,$

(J3) $\det A + \operatorname{tr} A + 1 > 0$ (always satisfied for $\beta,\rho\in(0,1)$, $\sigma,\kappa>0$).

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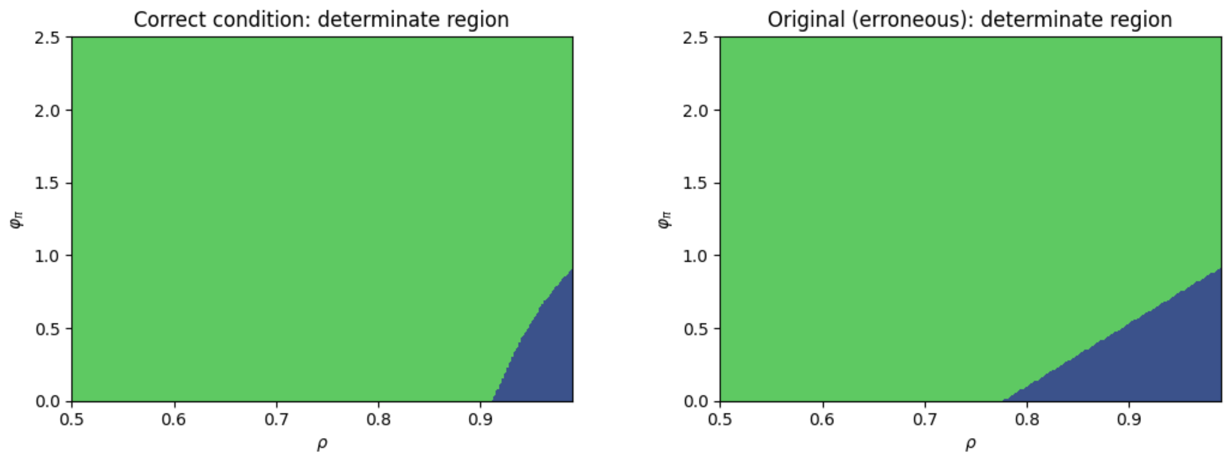
Given the restrictions on the parameter space, only J2 is relevant for assessing determinancy.

To get a sense of the difference, we plot determinancy regions using the correct inequality and the one from the paper. Both panels set: $\beta = 0.99$, $\sigma = 1$, $\kappa = 0.015$, and $\varphi_y = 0.05$, with the determinacy maps drawn over the grid $\varphi_\pi \in [0, 2.5]$ (vertical axis) and $\rho \in [0.5, 0.99]$ (horizontal axis).



(a) Correct condition $(1-\rho+\sigma\varphi_y)(1-\beta\rho)+\kappa\sigma(\varphi_\pi-\rho)>0$

(b) Appendix's original inequality $\varphi_\pi > \rho - \frac{1-\beta\rho}{\kappa}\varphi_y$

Figure 1: Determinacy (shaded) versus indeterminacy across ρ and φ_π.