Econ 6016: Problem Set 1

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*Due in class on Tuesday, February 18th.

Problem 1

Consider the AR(2) process

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \epsilon_t. \tag{1}$$

1. Show that a necessary condition for stationarity is that the coefficients lie inside the triangle

$$\phi_1 + \phi_2 < 1, \quad \phi_2 - \phi_1 < 1, \quad \text{and } \phi_2 > -1.$$

2. Now re-parameterize the AR(2) model in terms of partial autocorrelations ψ_1 and ψ_2:

$$\phi_1 = \psi_1(1 - \psi_2), \quad \phi_2 = \psi_2.$$

   Show that the AR(2) process is stationary if and only if $\psi_1 \in (-1, 1)$ and $\psi_2 \in (-1, 1)$.

Problem 2

Demonstrate that the first $p + 1$ Yule-Walker equations for the AR(p) process

$$y_t = \sum_{i=1}^{p} \phi_i y_{t-i} + \epsilon_t$$

are

$$\begin{aligned} \sigma_\epsilon^2 &= \gamma_{yy,0} - \sum_{i=1}^{p} \phi_i \gamma_{yy,i} \\ 0 &= \phi_i \gamma_{yy,0} - \gamma_{yy,i} + \sum_{j=1, j \neq i}^{p} \phi_j \gamma_{yy,|i-j|}, \quad i = 1, \ldots, p \end{aligned}$$

Calculate the first 3 autocovariances for the AR(3) process of Problem~2.

Problem 3

Download some aggregate time series from the Economic Database (FRED II) maintained by the Federal Reserve Bank of St. Louis: GDP (implicit price deflator), GDP Implicit price deflator, Real Personal Consumption Expenditure, Real Private Nonresidential Fixed Investment.

- Take logs of GDP, Consumption, and Investment. For each series, estimate a model of the form

$$y_t = \beta_1 + \beta_2 t + u_t$$

  using OLS.

- According to your estimates, what is the annualized average growth rate (in percent) of GDP, consumption, and investment?

- Compare the standard error estimates produced by the software that you are using for the OLS detrending to the asymptotic standard errors derived in class.

- Compute sample autocorrelation functions for the deviations of output, consumption, and investment (the $\hat{u}_t$'s) from their deterministic trend.

- Approximate growth rates of these 3 series by $\ln y_t - \ln y_{t-1}$ and compute the sample autocorrelation function for GDP growth. Compare the results to (i).

- Compute inflation rates as differences of the log GDP deflator. Compute the sample autocorrelation function for inflation.

- Repeat the above analysis for the subsamples "before 1970", "between 1970 and 1982", "after 1982". Did the persistence and the volatility of the series change?

- Repeat the above empirical analysis with four time series from some other industrialized country.

Reading McConnell, Margaret and Gabriel Perez-Quiros (2000): "Output Fluctuations in the United States: What has changed since the early 1980's?" *American Economic Review*, 90(5), 1464-76.

Problem 4

Consider the simple model:

$$y_t = \rho y_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_t^2), \quad |\rho| < 1. \tag{2}$$

Assume that σ_t^2 follows the process:

$$\sigma_t^2 = \begin{cases} \sigma^2 & : \quad t \text{ is even} \\ \alpha\sigma^2 & : \quad t \text{ is odd} \end{cases} \tag{3}$$

Consider the least squares estimator

$$\hat{\rho}_{LS} = \left(\sum_{t=2}^{T} y_{t-1}^2 \right)^{-1} \sum_{t=2}^{T} y_t y_{t-1}.$$

- Is $\hat{\rho}_{LS}$ a consistent estimator for ρ?
- Derive the asymptotic distribution of $\sqrt{T}(\hat{\rho}_{LS} - \rho)$ at $T \to \infty$. In particular what is the variance, $\mathbb{V}_{\hat{\rho}}$ of this distribution?
- Consider an estimator of the variance which assumes homoskedasticity:

  $$\frac{\frac{1}{T}\sum_{T=2}^{T} \epsilon_t^2}{\frac{1}{T}\sum_{T=2}^{T} y_{t-1}^2}$$

  What does this quantity converge in probability to? Call is $\mathbb{V}_{\hat{\rho}}^*$.
- What is the relationship between $\mathbb{V}_{\hat{\rho}}$ and $\mathbb{V}_{\hat{\rho}}^*$?

Problem 5

The Hodrick-Prescott filter (see lecture notes) has been criticized for amplifying the spectrum at certain business cycle frequencies. Consider a real business cycle model that is driven by a total factor productivity process of the form

$$y_t = \phi y_{t-1} + \epsilon_t, \quad \epsilon_t \sim WN(0, \sigma^2) \tag{4}$$

Let y_t^* be the HP detrended version of this productivity process. Derive and plot the spectrum of y_t and y_t^* for $\phi = 0.95$. Can you find any “spurious” cycles in the detrended data. What happens if ϕ decreases to 0.7?